Chapter 4

Noise Sources and Channel Impairments

There are various effects that can cause distortion of optical signal during modulation, propagation and detection processes, as illustrated earlier in Figure 1.11. In Chapter 3 we already discussed signal distortion caused by optical fiber loss, dispersion, and nonlinear effects. The impairments due to these effects will degrade and compromise the integrity of the signal before it arrives to the decision point in the optical receiver. In addition, there will be corruptive additives to the signal, that present a noise created in optical transmission channel, which will also come to the decision point mixed with the signal. The transmission quality is measured by the signal-to-noise-ratio (SNR), at the decision point. That ratio determines the receiver sensitivity, which is defined as a minimum optical power needed to keep SNR at the specified level. Received signal level at the decision point should be as high as possible to keep the distance between the signal and noise, and to provide a margin necessary to compensate for other corruptive effects.

In this chapter we describe the characteristics of the noise components in optical communication channel and evaluate their parameters. We will also evaluate the impairment parameters and determine the degradation of receiver sensitivity.

4.1 OPTICAL CHANNEL NOISE

The noise, as a major corruptive addition to the signal, can originate from different places within an optical transmission system, as shown in Figure 4.1. Semiconductor lasers are the source of the relative intensity noise (RIN), laser phase noise, and mode partition noise. Optical fibers are responsible for modal noise generation, while optical splices and optical connectors are the origin of the reflection-induced noise. Optical amplifiers generate spontaneous emission noise, which is subsequently amplified on the line and becomes so-called amplified spontaneous emission (ASE) noise. Also, there is a crosstalk that can be treated as a noise, which is generated mainly in multiplexing and switching network
elements (optical multiplexers, ROADM and OXC). Finally, there are several noise components generated during optoelectronic conversion in photodiode, all of them with either thermal or quantum nature.

All noise components mentioned above can be divided into ones generated at the optical level (optical noise), and the electrical noise components added to the signal while being in an electrical form. Any optical noise component carries some optical power, which is proportional to the square of its electric field. Since the photodetector generates a photocurrent that is proportional to the incoming optical power rather than to the incoming electric field associated with the power, there will be several side components of the total photocurrent. These components arise since the noise electric field beats against the signal field, against itself, and against the fields of other optical noise components. Although the number of beat components can be substantial, most of them are relatively small, and can be neglected.
original signal waveform. The impact of both the noise components and signal impairments can be evaluated at the output of preamplifier, which will be done in this chapter with respect to basic direct detection (DD) scheme. The advanced detection schemes will be discussed in Chapter 6.

![Diagram of noise components coming to the receiver front end](image)

**Figure 4.3 Noise components coming to the receiver front end**

The noise components coming to the receiver front-end can be sorted in a manner shown in Figure 4.3. Optical preamplifier is receiving the intensity noise and spontaneous emission noise from the transmission line. The spontaneous emission noise contains the component originating from last in-line amplifier, and the amplified spontaneous emission originating from the other amplifiers placed along the lightwave path. (The amplified spontaneous emission components from Figure 4.3 are denoted by lower case letters if they are related to one stage amplification, or by capital letters if they are related to multistage amplification.) Optical amplifier will enhance all optical inputs in proportion to the amplifier gain. Both signal and noise components are being converted to the electrical level through photodetection process. In addition to the components due to direct opto-electronic conversion, a several kinds of noise beat components have been created. The beating is caused by the interaction among electrical fields belonging to signal, spontaneous emission, and local laser oscillator (if a coherent detection scheme is deployed). Therefore, the total number of the beat noise components that come to the input of the front-end could be large, but just few of them will have a significant contribution to the overall noise strength. In case with direct detection, the most important noise beat components are the signal-spontaneous emission beat noise and the spontaneous emission-spontaneous emission beat noise. When local oscillator is present, intensity and phase noise components will be added.

There are has both multiplicative and additive components in the total noise [1], [2]. The multiplicative noise components are produced only if the signal is
present, while the additive components are present even if the optical signal is not
generated. The multiplicative noise components are:

- **Mode partition noise** generated in multimode lasers due to nonuniform
  distribution of the total power among longitudinal modes and microvariations
  in the intensity of each longitudinal mode.
- **Modal noise**, which arises in multimode fibers through the random process of
  excitation of transversal modes and the power exchange among them.
- **Laser intensity noise** caused by microvariations in the laser output power
  intensity. This noise is characterized through the relative intensity noise
  (RIN) parameter.
- **Laser phase noise** caused by microvariations in phase of generated photons.
  This is the main reason why the output optical signal, which is a collection of
  individual photons, exhibits a finite nonzero spectral width.
- **Quantum shot noise**, caused by quantum nature of light and the random
  distribution of the electrons generated during the photodetection process.
- **Avalanche shot noise**, caused by random nature of the amplification of
  primary electron-hole pairs through the effect of impact ionization in
  avalanche photodiodes.

The additive noise components are:

- **Dark current noise** generated in photodiodes caused by generation of the
  electron-hole pairs due to thermal process.
- **Thermal noise**, also known as Johnson noise, created in the resistive part of
  the input impedance of an optical receiver.
- **Amplified spontaneous emission (ASE) noise** that is generated by optical
  amplifiers placed along the lightwave path.
- **Crosstalk noise** that occurs in multichannel WDM systems when another
  signal interferes with the signal in question. The crosstalk is often analyzed
  separately since it is different in nature than other noise components. The
  main sources of the crosstalk noise are optical multiplexer/demultiplexers,
  and optical switching elements (ROADM and OXC).

We will analyze the impact of noise components mentioned above while paying
more attention to those components that are dominant in different detection
scenarios.

### 4.1.1 Mode Partition Noise

All noise components generated in semiconductor lasers are caused by
spontaneous emission of photons that accompanies the process of stimulated
radiation in the laser cavity. The spontaneous emission is a stochastic process and
has an impact to fluctuations of the output power parameters (intensity, phase, and
frequency). The spontaneously emitted photons supplement the coherent optical
power by contributing randomly to the signal amplitude and phase, thus causing
random perturbations in amplitude and phase of the output power. Accordingly,
these fluctuations in intensity and phase of the emitted light are the physical origin of the overall laser noise. Mode partition noise is associated with multimode Fabry-Perot semiconductor lasers, due to uncorrelated emission among longitudinal modes that are confined within the laser spectrum—please refer to Figure 2.10b. The difference in intensities of any pair of longitudinal modes from Figure 2.10b fluctuates randomly, even if we assume that the total output power is constant. These intensity fluctuations will be transferred all the way to the optical receiver since chromatic dispersion in the optical fiber will force all longitudinal modes to travel with different speeds. Consequently, the mode partition noise will be converted to the electrical noise, and will corrupt the signal at the decision point.

The impact of mode partition noise is more relevant for transmission systems where the product $B \cdot L$ is relatively low ($B$ is the signal bit rate, and $L$ is the transmission distance.) The intensive study of the mode partition noise was carried out in literature some time ago, in order to estimate the power penalty related to the noise impact [3], [4]. It was concluded that the impact of the mode partition noise could be almost entirely suppressed by satisfying the following condition

$$B \cdot L \sigma_{\lambda} \leq 0.075$$

where $D$ is the chromatic dispersion coefficient, while $\sigma_{\lambda}$ is the spectral linewidth of the multimode semiconductor laser—please see Table 2.1. The $B \cdot L$ product can be maximized by selecting an operational wavelength within zero dispersion region, or at the region where chromatic dispersion is lower than 1 ps/km·nm. As an example, the signals with 1 Gb/s bit rate can be transmitted over about 30 km, while 10 Gb/s signals can be effectively transmitted over 3 km.

Semiconductor lasers, such as DFB or DBR ones, designed to operate in single-mode regime do not produce mode partition noise. However, the existence of remaining side-modes in the laser spectrum may be of some concern. The strength of side-modes is characterized by the mode suppression ratio (MSR), which is defined as a difference in powers between the governing longitudinal mode and the most dominant suppressed side-mode, as shown in Figure 2.12b. We can assume that lasers with MSR > 100 will cause a negligible mode partition noise effect since the power penalty will be lower than 0.1 dB [3].

### 4.1.2 Modal Noise

In general case, the total input optical power in multimode optical fibers is nonuniformly distributed among a number of modes. Such modal distribution creates so-called speckle pattern at the receiving side, which contains brighter and darker spots in accordance with the mode distribution. The photodiode effectively eliminates the speckle pattern impact by registering the total power that is integrated over the photodiode area. However, if the speckle pattern is not stable but changes with time, it will induce fluctuations in the received optical power.
Such fluctuations are referred to as the modal noise, and will be eventually converted to photocurrent fluctuations. The fluctuations in speckle pattern occur in optical fibers due to mechanical disturbances, such as microbends and vibrations, within optical cables. Also, the splices and connectors will influence the power distribution over transversal modes since they act as spatial optical filters.

The modal noise is inversely proportional to the spectral linewidth $\Delta \nu$ of the light source. This comes from the fact that mode interference and speckle pattern changes are relevant only if coherence time $t_{coh}$ ($t_{coh} \sim 1/\Delta \nu$) is longer than intermodal dispersion in optical fibers—please refer to Section 2.2.3. This condition is not satisfied if light emitting diodes (LED) are used for signal transmission, since the LED spectral linewidth is quite large. Therefore, it is a good idea to use LED sources in combination with multimode optical fibers whenever possible to avoid the possible impact of modal noise. The situation is quite different if single mode lasers are used in combination with multimode optical fibers, since the modal noise impact could be quite a serious problem. The impact of the modal noise is higher for smaller number of modes propagating through optical fiber, while the most serious situation occurs if optical power at the receiving side is effectively shared by only several transversal modes, which is the case with few mode fibers and multicore fibers analyzed in Section 3.6. The numerical solution of coupled Equations (3.199)- (3.201), or Equation (3.226) with inclusion of the attenuation difference among the modes, can be used to analyze the impact of the modal noise to the performance of the transmission system. From system design perspective, it is necessary to allocate some power margin $\Delta P$ to accommodate the modal noise effect in case when single mode lasers are used in combination with multimode optical fibers, but without any advanced modulation/detection scheme in place. The allocated margin, which effectively means that signal optical power should be increased by $\Delta P$ in order to counterattack the impact of the modal noise, should be as high as 1 dB for combination of single mode lasers and multimode optical fibers.

In addition to multimode fibers, even sections of single mode optical fibers that are up to a few meters long can introduce the modal noise, since a higher order mode can be excited at the fiber discontinuity (connector or splice), and then converted back to the fundamental mode at the next discontinuity. It is good idea, therefore, to use optical fibers that are a little longer than necessary even if the distance is just 1-2 meters. The distance of 5 meters, for example, can effectively eliminate the impact of the modal noise since higher-order mode cannot reach the second fiber discontinuity. It is important to notice that vertical cavity surface emitting lasers (VCSEL) are often used in combination with multimode optical fibers for very short links (up to several kilometers). Although this combination is a cost effective solution for gigabit signal rates over very short distances, it is also a place where modal noise may be a serious factor causing power penalty even higher than 1 dB [5].
4.1.3 Laser Phase and Intensity Noise

The total power of the quantum noise generated in semiconductor lasers through process of stimulated emission can be evaluated by multiplying the density of the photons representing the noise with the energy of a single photon. The photon density number of the quantum noise is represented by the spontaneous emission factor $n_{sp}$ expressed through the electron populations $N_2$ and $N_1$ at the upper and lower energy levels, as shown in Equation (2.15), so we have that

$$W_{qn} = n_{sp}hv = \frac{N_2}{N_2 - N_1}hv$$

(4.2)

where $W_{qn}$ is the total energy of the quantum noise per longitudinal mode in semiconductor lasers. The total power $P_{qn}$ of the laser quantum noise is calculated as

$$P_{qn} = W_{qn}\Delta\nu = n_{sp}hv\cdot\Delta\nu$$

(4.3)

where $\Delta\nu$ is laser spectral bandwidth. In addition to quantum noise, the total noise generated in semiconductor lasers contains the thermal component as well. The thermal component $P_{tn}(\nu)$ of the laser mode is calculated by dividing the Planck’s Equation (10.9) by the number of laser eigenmodes $N_{emod} = \frac{8\pi\nu^2}{c^3}$, [7], so we have that

$$P_{tn} = \frac{h\nu\Delta\nu}{\exp(h\nu/k\Theta) - 1}.$$  

(4.4)

and

$$P_{tn} = \left[ \frac{N_2hv}{N_2 - N_1} + \frac{hv}{\exp(h\nu/k\Theta) - 1} \right] \Delta\nu$$

(4.5)

where $P_{tn}(\nu)$ is the total noise generated per laser mode.

Since the laser noise is a random process, both the laser phase and intensity noise components can be evaluated by using standard methods of signal analysis [2]. It is necessary to find auto-correlation function, cross-correlation function, and power spectral density with respect to the amplitude and phase of generated light. For that purpose the electric field of the generated light can be expressed as [8]

$$E(t) = [A_0 + a_r(t)]\exp[j(\phi_0 + \phi_r(t))]$$

(4.6)
where $A_0$ and $\phi_0$ are the stationary values of the amplitude and the phase, respectively, while $a_n(t)$ and $\phi_n(t)$ are of noisy fluctuations of these values.

The phase fluctuations $\phi_n(t)$ define the spectrum of frequency noise $S_f(\nu)$ since the instantaneous frequency deviations are defined as

$$\nu_n(t) = \frac{1}{2\pi} \frac{d\phi_n}{dt}$$  \hspace{1cm} (4.7)$$

The spectral density $S_f(\nu)$ of the frequency noise is defined as

$$S_f(\nu) = \lim_{T \to \infty} \frac{1}{T} \left\langle F_n(\nu) F^*_n(\nu) \right\rangle$$  \hspace{1cm} (4.8)$$

where $F_n(\nu)$ is the Fourier transform of the function $\nu_n(t)$.

In addition to spectral density function defined by Equation (4.8), it is necessary to find spectral density function of the electric field itself in order to evaluate the phase noise spectrum generated in the laser. The spectral density function can be found as Fourier transformation of the autocorrelation function $R(\tau)$ of the electric field [8], and by using Equations (4.6) and (4.8), so we have that

$$R(\tau) = \frac{\left\langle E_n(t)E_n(t+\tau) \right\rangle}{A_0^2} = \left\langle \exp j\Delta \phi_n(\tau) \right\rangle = \exp \left\{ -\frac{\left\langle \Delta \phi_n^2(\tau) \right\rangle}{2} \right\}$$  \hspace{1cm} (4.9)$$

where $\Delta \phi_n(\tau) = \phi_n(t+\tau) - \phi_n(t)$ defines phase fluctuations. It is assumed that phase fluctuations can be expressed as a Gaussian random process.

The Fourier transformation of Equation (4.9) and functional relations (4.7), and (4.8) lead to the following

$$\left\langle \Delta \phi_n^2(\tau) \right\rangle = \sigma^2_\phi = 4 \int_0^\infty S_f(\nu) \sin^2 \left( \frac{\nu \tau}{2} \right) d\nu$$  \hspace{1cm} (4.10)$$

where $\sigma^2_\phi$ is the phase noise variance that is often used in system calculations.

Spectral density $S_f(\nu)$ has been evaluated in number of papers, [9] to [13], for different laser structures. The following expression for InGaAsP laser structure were obtained in [10]

$$S_f(\nu) = \frac{n_e \hbar \nu \Delta \nu_{LR}}{A_0^2} \left[ 1 + \frac{\alpha^2_{\text{chirp}} \nu^4}{(\nu^2 - \nu_{\text{RF}}^2)^2 + \nu^2 \left( \frac{\Delta \nu_{\text{RF}}}{\nu} \right)^2} \right],$$  \hspace{1cm} (4.11)$$
where $\alpha_{chirp}$ is an amplitude-phase coupling parameter defined by Equation (2.20), $\Delta v_{chirp}$ is the frequency bandwidth of the laser resonator, while $v_R$ and $\gamma_s$ are the relaxation frequency and depletion constant, respectively, defined as

$$v_{chirp}^2 = \frac{\alpha_{chirp} G(P_0) \delta G}{(2\pi)^2 \delta P}$$

$$\gamma_s = \left( \frac{\alpha_{chirp}^2}{\alpha_{chirp} + 1} + \alpha_{chirp}^2 \frac{\delta G}{\delta P} \right)^{1/2}$$

where $P$ is the radiated optical power measured by the number of photons involved in the process, $P_0$ is the initial number of photons, $G$ is the net gain related of the stimulated emission—please refer to Equations (2.13) and (2.18).

The laser intensity noise, which has the same nature as the mode partition noise, is associated with single mode lasers. The intensity fluctuations created at the transmitter side will eventually experience both the attenuation in the optical fiber and amplification through the chain of optical amplifiers. The laser intensity noise will be converted to the electrical noise by a photodiode and corrupt the signal at the decision circuit point.

Laser intensity noise can be estimated by relative intensity noise (RIN) parameter, which is Fourier transform of the intensity autocorrelation function $\Phi(\tau)$ defined as

$$\Phi(\tau) = \frac{\langle \delta P(t) \delta P(t+\tau) \rangle}{\langle P \rangle^2}$$

where $\langle P \rangle$ presents the average value of the laser output power measured by number of created photons, while $\delta P = P(t) - \langle P \rangle$ presents small power fluctuations around the average value. It is therefore

$$RIN(v) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(\tau) \exp(-j2\pi v \tau) d\tau = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\langle \delta P(t) \delta P(t+\tau) \rangle}{\langle P \rangle^2} \exp(-j2\pi v \tau) d\tau$$

(4.15)

The RIN value can be calculated by solving generalized laser rate equations that contain Langevin noise term $F_{\ell}(t)$ related to fluctuations of the intensity—please see Equations (2.21-2.23). The parameter $RIN(v)$, which is usually expressed in decibels per Hertz, has a peak maximum at relaxation-oscillation frequency $v_R$.

The parameter of practical interest that measures the impact of the intensity noise is defined as

$$\epsilon_{int}^2 = \left[ \frac{\langle \delta P(t) \delta P(t+0) \rangle}{\langle P \rangle^2} \right] = \int_{-\infty}^{\infty} RIN(\omega) d\omega = 2RIN_{low} \Delta f$$

(4.16)
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where $\Delta f$ defines the bandwidth applicable to the intensity noise (which is in fact the bandwidth of an optical receiver), while $RIN_{\text{laser}}$ is parameter that characterizes magnitude of the intensity noise. It is $RIN_{\text{laser}} \sim -160 \text{ dB/Hz}$ for high quality DFB lasers.

The reflection-induced noise is caused by the appearance of back-reflected optical signal due to refractive-index discontinuities at optical splices, connectors, and optical fiber ends. It has the same nature as the laser intensity noise, and that is the reason why they are often treated together. The power of the reflected light can be estimated by the reflection coefficient $r_{\text{ref}}$, defined as

$$r_{\text{ref}} = \left| \frac{n_a - n_b}{n_a + n_b} \right|^2$$

(4.17)

where $n_a$ and $n_b$ are the refractive-index coefficients of materials facing each other. The amount of the reflected light is directly proportional to the coefficient $r_{\text{ref}}$. Therefore, it is higher for bigger difference in refractive indexes, and vice versa. The strongest reflection occurs at the glass-air interface. We can assume that $n_a = 1.46$ (for silica), and $n_b = 1$ (for the air), which returns the reflection coefficient $r_{\text{ref}} \sim 3.5\%$ (or $-14.56 \text{ dB}$). This value can be even higher if optical fiber ends are polished. The amount of reflected light can be reduced below 0.1% if some index-matching oils or gels are used at the fiber-air interface, or if the fiber ends are cut at an angle to deviate the reflected light from the fiber axis. Both methods are extensively used in high-speed optical transmission systems. A considerable amount of back-reflected light can come back and enter semiconductor laser resonant cavity, which would negatively affect the laser operation and lead to the excessive intensity noise at the laser output. That is the main reason why the laser is commonly separated from the optical fiber link by an optical isolator, which will eventually suppress the impact of the reflected light. The relative intensity noise can be increased by as much as 20 dB if the back-reflected light exceeds 30 dBm level.

The impact of the reflection-induced noise is not limited just to the laser source, since multiple back and forth reflections between optical splices and connectors can be the source of an additional intensity noise. Multiple reflections will eventually create multiple copies of the same signal traveling forward. These copies will be shifted in phase and act as a phase noise. Such phase noise is eventually converted to the intensity noise by chromatic dispersion, and enhanced by optical amplifiers along optical fiber links. In addition to chromatic dispersion, phase noise can be converted to intensity noise at any two reflecting surfaces along the optical fiber links, since they act as the mirrors of Fabry-Perot interferometer. The end result of conversion of the phase noise to intensity noise will be an increase of the total relative intensity noise. Therefore, it is extremely important to suppress the back-reflections along the entire optical transmission line by careful selection of optical connectors that minimize reflections.
4.1.4 Quantum Shot Noise

The optical signal coming to photodiode contains a number of photons that would generate the electron-hole pairs by the photoelectric effect. The electron-hole pairs are effectively separated by the inverse bias voltage, which produces a photocurrent, as illustrated in Figure 10.2. The probability of having \( n \) electron-hole pairs at the photodiode during the time interval \( \Delta t \) is expressed by the Poisson probability distribution [2], [14],

\[
p(n) = \frac{N^n e^{-N}}{n!}
\]

(4.18)

where \( N \) is the mean number of photoelectrons detected during the time interval \( \Delta t \), given as

\[
N = \frac{\eta N}{h v} P(t) \, dt
\]

(4.19)

The parameters in Equation (4.19) are: \( v \) – optical frequency; \( h \) - the Plank’s constant; \( \eta \) - the quantum efficiency, which is the ratio between the number of the electrons detected and the number of the photons that arrived, as presented by Equation (2.115). The Poisson distribution approaches a Gaussian distribution for larger values of the mean \( N \).

The mean intensity of the photocurrent, which has been generated by the stream of electrons, is given as

\[
I = \langle i(t) \rangle = \frac{q N}{\Delta t} = \frac{q N}{T}
\]

(4.20)

where \( q \) is the electron charge (\( q = 1.6 \times 10^{-19} \) Coulombs.) Please notice that it was assumed that the time interval \( \Delta t \) is equal to the duration \( T \) of the either “1” or “0” bits. The actual number of electrons generated during the bit duration will fluctuate around the mean value \( N \) due to a random nature of the photodetection process, while the generated photocurrent will fluctuate around the mean value \( I \). Due to the property of Poisson distribution that the variance is equal to the mean value, we have that

\[
\langle [n - N]^2 \rangle = N
\]

(4.21)

where the angle brackets indicate averaging function. The instantaneous current during the bit duration \( T \) is given as
Equations (4.21) and (4.22) can be used to estimate the fluctuations of the instantaneous current around the mean value. These fluctuations can be expressed through the mean-square value, which is

\[
\langle i^2 \rangle_{\text{in}} = \langle [i(t) - I]^2 \rangle = \frac{q^2 \sigma^2}{T^2} = \frac{q^2 N}{T^2} = \frac{qI}{T} \tag{4.23}
\]

Equation (4.23) presents the power of the quantum shot noise, which is a multiplicative noise component caused by the quantum nature of light. We can correlate the bit duration \( T \) with the signal bandwidth \( \Delta f \) by assuming that \( \Delta f = 1/2T \) as in [15] to get the following relation

\[
\langle i^2 \rangle_{\text{in}} = 2qI\Delta f = S_{\text{sn}}(f)\Delta f \tag{4.24}
\]

The same result as previous one can be obtained by calculating the signal spectral density \( S_{\text{sn}}(f) \) through the Fourier transform of the photocurrent correlation function [16], [17]. The spectral density of the quantum shot noise from Equation (3.23) is constant and given as

\[
S_{\text{sn}}(f) = \frac{d}{df} \langle i^2 \rangle_{\text{in}} = 2qI \tag{4.25}
\]

Equations (4.24 - 4.25) can be applied just to PIN photodiodes. That is because an internal amplification process in avalanche photodiodes (APD) increases the generated photocurrent and enhances the total quantum noise. Physical background behind this additional noise in APD is related to the fact that secondary electron-hole pairs are generated randomly through stochastic process of impact ionization.

It was shown that the avalanche shot noise can be characterized by the Gaussian probability density function, and has the spectrum that is flat with the frequency [15]. The avalanche shot noise power is

\[
\langle i^2 \rangle_{\text{in,APD}} = S_{\text{sn,APD}}(f)\Delta f = 2q\langle M \rangle^2 F(M)I\Delta f \tag{4.26}
\]

where \( \langle M \rangle \) is the average value of the avalanche gain, while \( F(M) \) is the excess noise factor, which measures the variations of the instantaneous avalanche gain \( M \) around its average value. The amplification process is always noisy since we have that \( F(M) > 1 \). Therefore, the increase of the noise will be proportionally higher
than the signal enhancement. The excess noise factor can be expressed as [15], [16]

\[ F(M) = k_N \langle M \rangle + (1 - k_N) \left( 2 - \frac{1}{\langle M \rangle} \right) \]  

(4.27)

where parameter \( k_N \) is known as the ionization coefficient. The ionization coefficient takes the values in the range from 0 to 1, and measures the ability of a carrier to generate other carriers in the avalanche amplification process. This coefficient should be as small as possible in order to minimize the avalanche shot noise that is generated. An approximate form, which is very often used instead of Equation (4.27), is given as

\[ F(M) = \langle M \rangle^x \]  

(4.28)

The noise coefficient \( x \) in Equation (4.28) takes the values in the range 0 to 1, depending on the semiconductor compound used. Typical values of the noise coefficients \( k_N \) and \( x \) for commonly used APD compounds are shown in Table 4.

<table>
<thead>
<tr>
<th>Semiconductor</th>
<th>( x )</th>
<th>( k )</th>
<th>Dark current noise in nA</th>
</tr>
</thead>
<tbody>
<tr>
<td>InGaAs</td>
<td>0.5-0.8</td>
<td>0.3-0.6</td>
<td>Up to 20</td>
</tr>
<tr>
<td>Germanium</td>
<td>1.0</td>
<td>0.7-1.0</td>
<td>50-500</td>
</tr>
<tr>
<td>Silicon</td>
<td>0.4-0.5</td>
<td>0.02-0.04</td>
<td>Up to 10</td>
</tr>
</tbody>
</table>

In order to evaluate the impact of the shot noise, we will assume that noise spectral density for both PIN and APD has a form

\[ S_n(f) = 2q\langle M \rangle^{2+x} I \]  

(4.29)

Example 1: If an optical power of \( P = -20 \, dBm \) falls to PIN photodiode with responsivity \( R = 0.8, M = 1 \), and \( x = 0 \), it will produce \( I = 8 \, \mu A \) and \( S_{n,PIN} \sim 2.6 \times 10^{-24} \, A^2/Hz \).

Example 2: If an optical power of \( P = -20 \, dBm \) falls to APD with responsivity \( R = 0.8, M = 10 \), and \( x = 0.7 \), it will produce \( I = 80 \, \mu A \) and \( S_{n,APD} \sim 1.29 \times 10^{-21} \, A^2/Hz \).

4.1.5 Dark Current Noise

The dark current consists of electron-holes pairs, which are thermally created in the photodiode p-n junction and flow through a biased photodiode even if no light is coming to the photodiode surface. These carriers can also get accelerated in
APD, and can contribute to the avalanche shot noise generation. The dark current noise power is

\[
\langle i^2 \rangle_{\text{dcn}} = S_{\text{dcn}}(f)\Delta f = 2q(M)^2 F(M)I_d \Delta f
\]

(4.30)

where \(I_d\) is the primary dark current in photodiode, while \(S_{\text{dcn}}\) is spectral density of the dark current. The typical values of dark current for different semiconductor compounds are shown in Table 2.3. Equation (4.30) can be applied to both PIN photodiode and APD.

**Example 1:** If \(I_d = 5\) nA, \(M = 1\), and \(x = 0\) for PIN photodiode, we have that \(S_{\text{dcn}} \approx 1.6 \times 10^{-27}\) A\(^2\)/Hz.

**Example 2:** If \(I_d = 5\) nA, \(M = 10\), and \(x = 0.7\) for APD photodiode, which will produce the spectral density \(S_{\text{dcn}} \approx 8 \times 10^{-25}\) A\(^2\)/Hz.

As we can see, the power of dark noise generated in photodiodes is smaller than the power of the other noise components that can be generated during photodetection. That is the reason why the impact of dark current noise is sometimes neglected.

The process of the overall noise generation in the avalanche photodiode is illustrated in Figure 4.4. There are four time slots in Figure 4.4 to illustrate the Poisson statistics and the total noise generation. There is one incoming photon that generates one primary photoelectron within the first time slot. This primary electron will produce several secondary electron-hole pairs (it is seven in Figure 4.4). There is no signal photons captured within the second time slot, but a dark
current electron is generated, and it was able to produce a pair of secondary electrons through the ionization process. Next, there are two incident photons within the third time slot that produce a number of secondary electrons. Finally, the time slot number four is similar to the slot number two, with the exception that a smaller number of secondary electrons have been generated. As a result, both the total number of electrons and generated current flow fluctuates in time around their average value. These fluctuations are associated with the current noise at the photodiode output.

### 4.1.6 The Thermal Noise

The load resistor, which is used to convert the photocurrent to voltage, as shown in Figure 2.32, generates its own noise due to a random thermal motion of electrons. Such noise occurs as a fluctuating current that adds to the generated photocurrent. This additional noise component, also known as Johnson noise [18], has a flat frequency spectrum that is characterized with the zero-mean Gaussian probability density function. This spectral density is expressed in $\text{A}^2/\text{Hz}$, and given as

$$S_{\text{th}}(f) = \frac{4k\Theta}{R_L}$$  \hspace{1cm} (4.31)

where $R_L$ is the load resistance, $\Theta$ is the absolute temperature in Kelvins, while $k$ is the Boltzmann’s constant ($k = 1.38 \cdot 10^{-23} \text{J/K}$). The thermal noise power contained in the receiver bandwidth $\Delta f$ is

$$\langle i^2 \rangle_{\text{th}} = S_{\text{th}}(f) \Delta f = \frac{4k\Theta\Delta f}{R_L}$$  \hspace{1cm} (4.32)

The thermal noise can be reduced by using a large value load resistance. Such design, often referred as the high-impedence front-end amplifier, also increases the receiver sensitivity. On the other side, it limits the receiver bandwidth since the $RC$ constant ($C$ is the capacitance of the circuit) is also increased. Therefore, the high-impedance input requires an equalizer that will boost the high frequency components and increase the receiver bandwidth. In general case, the receiver bandwidth is increased by selecting smaller value of the load resistance. The design with low-impedance front-end has a smaller receiver sensitivity that the design with the high-impedence front-end amplifier. As a compromise, the trans-impedance front-end, shown in Figure 2.32, is used to achieve both high receiver sensitivity and high-speed operation. The trans-impedance front-end design also improves the dynamic range of the optical receiver, which is important in cases when significant variations of optical power can occur at the receiving side.
The thermal noise generated in the load resistor will be enhanced by electronic components within the front-end amplifier. That noise contribution can be accounted for by the amplifier noise figure $NF_{ne}$, which is the factor that measures the thermal noise enhancement at the front-end output. The total power of the thermal noise that also accounts the contribution of the front-end amplifier is given as

$$S_{th} = \frac{4k\Delta f \cdot NF_{ne}}{R_L} = I_{th}^2 \cdot NF_{ne}$$

The noise figure $NF_{ne}$ (the index “e” stands for “electrical”) can vary from amplifier to amplifier, but for a low noise front-end amplifier it is around 3 dB. The parameter $I_{th}$, which is expressed in A/Hz$^{1/2}$, is equivalent to standard deviation of the thermal current. This parameter is usually several pA/Hz$^{1/2}$. Parameter $q$ in the previous equation represents the electron charge ($q = 1.6 \cdot 10^{-19}$ C), $k$ is the Boltzmann’s constant ($k = 1.38 \cdot 10^{-23}$ J/K), $\Theta$ is the absolute temperature in Kelvins, $R_L$ is a load resistance in ohms. By assuming that $I_{th}=3$ pA/Hz$^{1/2}$, and $NF_{ne}= 2$, we can obtain that $S_{th}= 2 \cdot 10^{-23}$ A$^2$/Hz. Therefore, calculated value for spectral density of the thermal noise is at least about two orders of magnitude higher than the one related to the dark current noise. The shot noise component produced in PIN photodiode will be at the same level as the thermal noise component if optical power of $-10.65$ dBm ($86 \mu$W) arrives at the photodiode with responsively $R=0.8$ A/W.

### 4.1.7 Spontaneous Emission Noise

Signal amplification in an optical amplifier, which is discussed in Section 2.6, is also accompanied by spontaneous emission of the photons. That process is additive, which means that there is no correlation between the signal and the noise generated through the spontaneous emission. The noise induced by spontaneous emission has also a flat frequency spectrum characterized with the zero-mean Gaussian probability density function. The noise spectral density can be written as [19]

$$S_{sp}(\nu) = (G-1)NF_{no}h\nu/2$$

where $G$ is the optical amplifier gain, $NF_{no}$ is optical amplifier noise figure that measures the noise increase (the index “o” stands for “optical”), $h$ is the Planck’s constant ($h=6.63 \cdot 10^{-34}$ J/Hz), while $\nu$ is the optical frequency. Please notice that we will temporarily carry two notations for frequency i.e. $f$ for the frequency of an electrical signal, and $\nu$ for the frequency of an optical signal. However, the variables $f$ and $\nu$ refer to the same physical parameter, which is expressed in Hertz’s. This distinction will be used for few more times (for example to
distinguish optical and electrical filter bandwidths, and optical and electrical signal to noise ratios.)

There is the following relation between the noise figure and the spontaneous emission factor 
\[ n_{sp} = \frac{(N_1 - N_2)}{N_2} \], which is the same parameter as one defined in Equations (2.15) and (4.2)

\[ NF_{no} = \frac{2n_{sp}(G-1)}{G} \approx 2n_{sp} \geq 2 \]  \hspace{1cm} (4.35)

The populations \( N_1 \) and \( N_2 \) are related to the number of electrons at the ground level and at the upper energy level, respectively—please refer to Figure 2.26a. Theoretically, the spontaneous emission factor will become unity if all electrons are moved in energy to the upper level, which is not possible in practice. That is the reason why the spontaneous emission factor will always take values higher than one. In most practical cases it will be in the region from 2 to 5, which corresponds to 3-7 dB. The effective noise figure of the amplifier chain of cascaded optical amplifiers can be calculated as

\[ NF_{n_{no,eff}} = NF_{n_{no,1}} + \frac{NF_{n_{no,2}}}{G_1} + \frac{NF_{n_{no,3}}}{G_1G_2} + \ldots + \frac{NF_{n_{no,k}}}{G_1G_2\ldots G_{k-1}} \]  \hspace{1cm} (4.36)

where \( NF_{n_{no,eff}} \) is the effective noise figure of the amplifier chain that contains the total number of \( k \) optical amplifiers. The first amplifier in the chain is the most important one in terms of the noise impact. That is the reason why multistage optical amplifiers should be designed to have the first stage with lower noise figure. Accordingly, any decrease in the effective value of the amplifier’s noise figure will bring a significant benefit to the overall system performance.

The total power of the spontaneous emission noise can be calculated as

\[ P_{sp}(v) = 2|E_{sp}|^2 = 2S_{sp}(v)B_{sp} = (G-1) \cdot NF_{n_{no}} h v B_{sp} \]  \hspace{1cm} (4.37)

where \( E_{sp} \) is the electric field of the spontaneous emission, while \( B_{sp} \) is the effective bandwidth of spontaneous emission determined either by the optical amplifier bandwidth, or by the optical filter. Please notice that factor 2 in Equation (4.37) accounts for contributions of two fundamental polarization modes that are present at the output of the optical amplifier.

4.1.8 Beating Components of the Noise in Optical Receiver

If there is a chain of optical amplifiers, the spontaneous emission noise generated in a preceding amplifier stage is eventually amplified in the following stage, thus becoming the amplified spontaneous emission (ASE) noise. In addition, the spontaneous emission noise is also generated at the any specific amplifier in
question, which means that the amplifier output is noisier than the input, as illustrated in Figure 4.3. The power of the amplified spontaneous emission noise is being converted from optical to electrical level in parallel with the optical signal conversion. The total photocurrent generated at the photodiode output, in case when optical amplifiers are employed, can be written as

\[ I_p = I + i_{\text{noise}} = G \sqrt{P} + E_{\text{sp}} + i_{\text{in}} + i_{\text{he}} \]  \hspace{1cm} (4.38)

where \( E = (P)^{1/2} \) and \( E_{\text{sp}} = (P_{\text{sp}})^{1/2} \) are the electrical fields associated with optical signal power \( P \) and amplified spontaneous emission power \( P_{\text{sp}} \), respectively. \( I \) is the signal current \((I = RP)\), \( R \) is photodiode responsivity, \( G \) is the amplifier gain, \( i_{\text{in}} \) is the quantum shot noise component, and \( i_{\text{he}} \) is the thermal noise component. The evaluation of noise components in most typical detection scenarios has shown that the three noise components from Equation (4.38) are the dominant ones, and the most relevant from the system design perspective.

The amplified spontaneous emission (ASE) is not simply converted to the corresponding electrical noise since there is a beating process between the ASE and the signal electric fields, which results in appearance of several components that can be classified as the beat noise components. It is, therefore, necessary to express the incoming signal and ASE by the corresponding electric fields, as in Equation (4.38), in order to evaluate these beat noise components. Please notice that Equation (4.38) contains just a half of the noise power from Equation (4.37). It belongs to a component of the ASE that has the same polarization with the signal. This is because the orthogonally polarized components cannot beat effectively, and only the component that has the same polarization with the signal is a relevant factor. The total noise current associated with the optical ASE noise occurs due to beating of the ASE field \( E_{\text{sp}} \) with signal field \( E \), and due to beating of the field \( E_{\text{sp}} \) with itself. The total variance of such a fluctuating current can be found by expressing all electrical fields in Equation (4.38) through the optical power by using a general expression \( E = \sqrt{P} \exp(-j\omega t) \), and by averaging the field products over random phases as in [10-11]. This process leads to the equation

\[ \langle i^2 \rangle = \langle i_{\text{he}}^2 \rangle + \langle i_{\text{in}}^2 \rangle + \langle i_{\text{sp}}^2 \rangle + \langle i_{\text{sp,op}}^2 \rangle \]  \hspace{1cm} (4.39)

where the first term on the right side represents the power of the thermal noise, while the three remaining terms have the following values

\[ \langle i_{\text{he}}^2 \rangle = \frac{4kT \cdot NF \cdot \Delta f}{R_t} \]  \hspace{1cm} (4.40)

\[ \langle i_{\text{in}}^2 \rangle = 2qR[GP + S_{B_{\text{op}}} \Delta f] = S_{m,\text{in}} \Delta f \]  \hspace{1cm} (4.41)

\[ \langle i_{\text{sp}}^2 \rangle = 4R^2GPS_{B_{\text{op}}} \Delta f = S_{\text{sp}} \Delta f \]  \hspace{1cm} (4.42)
\[
\langle i \rangle_{sp-sp} = 2R^2 S^2 / (2B_{sp} - \Delta f) \Delta f = S_{sp-sp} \Delta f 
\] (4.43)

The total spectral density of the beat noise components from Equations (4.42) and (4.43) can be expressed as

\[
S_{\text{beat}}(f) = S_{\text{sig-sp}}(f) + S_{sp-sp}(f) 
\] (4.44)

As we can see, the optical amplifier gain coefficient \( G \), bandwidth of the optical filter \( B_{op} \), and bandwidth of an electrical filter of the receiver \( \Delta f \) will play the key role in the size of the total noise caused by spontaneous emission in optical amplifiers. It is also important to notice from Equation (4.41) that the shot noise is higher when optical signal is preamplified.

Example: Let us assume the following typical values of the amplifier and photodiode parameters: \( P = -20 \, \text{dBm} \), \( G = 100 \), \( F_{no} = 3.2 \) (which is 5 dB), \( R = 0.8 \, \text{A/W} \), \( B_{op} = 0.1 \, \text{nm} \), \( \Delta f = 0.5 \, B_{op} \). The spectral densities from Equations (4.41), (4.42) and (4.43) now become

\[
S_{\text{sig-sp}} \approx 1.05 \times 10^{-19} \, \text{A}^2/\text{Hz}, \\
S_{sp-sp} \approx 0.53 \times 10^{-22} \, \text{A}^2/\text{Hz}, \\
S_{\text{sn,Amp}} \approx 2.5 \times 10^{-22} \, \text{A}^2/\text{Hz}, 
\]

As we can see, the beating noise components are larger than the shot noise component in PIN photodiodes.

### Table 4.2

Typical values of spectral densities associated with noise components

<table>
<thead>
<tr>
<th>Noise spectral density [A^2/Hz]</th>
<th>PIN</th>
<th>APD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dark current</td>
<td>1.6 \times 10^{-27}</td>
<td>1.6 \times 10^{-27}</td>
</tr>
<tr>
<td>Thermal noise</td>
<td>2 \times 10^{-24}</td>
<td>2 \times 10^{-24}</td>
</tr>
<tr>
<td>Shot noise without preamp</td>
<td>2.6 \times 10^{-24}</td>
<td>2.6 \times 10^{-25}</td>
</tr>
<tr>
<td>Shot noise with preamp</td>
<td>2.6 \times 10^{-22}</td>
<td>2.6 \times 10^{-23}</td>
</tr>
<tr>
<td>Signal-ASE beat noise</td>
<td>1.04 \times 10^{-23}</td>
<td>1.04 \times 10^{-24}</td>
</tr>
<tr>
<td>ASE-ASE beat noise</td>
<td>0.54 \times 10^{-22}</td>
<td>0.54 \times 10^{-24}</td>
</tr>
</tbody>
</table>

When considering the impact of individual noise components, we should differentiate two cases that are related to detection of “1” and “0” bits, respectively. That distinction is important with respect to levels of the shot noise, the beating noise components, and intensity related noise components. The extinction ratio can be used as a measure of the difference in receiving power associated with “1” and “0” bits. The comparison of different noise components that are related to “1” and “0” bits it is shown in Table 4.2 for both PIN photodiode and APD. It was assumed that the extinction ratio in accordance with the Equation (2.57) is 10. As we see from Table 4.2, thermal noise and the shot noise contributions to the total noise power are generally smaller than contribution...
coming from the beat noise components. It is possible to reduce the spontaneous-
spontaneous beat noise by optical filtering, and make it smaller than the signal-
spontaneous beat noise.

4.1.9 The Crosstalk Noise

Signal crosstalk occurs in multichannel systems when fraction of unwanted signal
adds to the power of the channel in question, thus acting as a noise. The crosstalk
noise can have either out-of-band or in-band nature.

The out-of-band or intrachannel crosstalk occurs when the power from a
neighboring channel crosses the border between channels and mixes with the
power of the specified optical channel, as shown in Figure 4.5a. This case happens
very often when optical filters and optical multiplexers are deployed within an
optical transmission system. Optical receiver bandwidth captures the interfering
power and converts it to the electrical current. Out-of band crosstalk is different
from the random noise in an advantageous way. Namely, various noise types have
amplitude probability distribution with a gradually decaying tail, which
determines the decision threshold position. On the other side, in crosstalk case the
undesired crosstalk power that may appear at the decision point is bounded since
there are a finite number of contributing sources. The worst case in terms of
crosstalk impact would be if all channels are bit-synchronized, and when channel
in question carries “0” bit while all other channels carry “1” bits. However, in
reality, the incoming crosstalk optical power is not correlated with the channel
signal, which means that the out-of band noise is incoherent in nature. The out-of-
band crosstalk is dominated by two immediately adjacent channels, as illustrated
in Figure 4.5a.

The photocurrent produced as a result of conversion of the out-of-band
signal can be considered as a noise, and treated similarly
as the way dark current noise was treated. The noise current generated by
conversion of the out-of-band crosstalk to the electrical signal is

\[ I_{\text{cross, out}} = \sum_{m=0}^{M} RX_n \]  \hspace{1cm} (4.45)

where \( R \) is photodiode responsivity, while \( X_n \) is portion of the \( n-th \) channel power
that has been captured by the optical receiver of the \( m-th \) optical channel (i.e.
channel in question). We assumed that there is \( M \) optical channels altogether, and
that the largest contribution to the crosstalk noise level comes from the
neighboring channels. The impact of the out-of-band crosstalk can be minimized
by optimizing the optical channel spacing and by selecting optical filters with
steeper bandwidth characteristics.
The in-band, or interchannel crosstalk, occurs when a signal at the same optical wavelength as the wavelength of the channel in question comes at the optical receiver to be converted to electrical current. This can happen if there are multiple optical multiplexing/demultiplexing stages between optical transmitter and optical receiver, or if an optical routing takes the place along the lightwave path. Any separation of optical channels by optical filtering process can cause interchannel crosstalk. That crosstalk can be enhanced by an increased mismatch between multiplexing and demultiplexing filters, which is usually caused by the temperature change, or by aging. This will create out-of-band noise, as mentioned above, if demultiplexing is followed by photodetection process.

However, if the signal continues its propagation along a specified lightpath, the total power will now consist of the signal power and out-of-band crosstalk power. Any additional multiplexing/filtering process that can eventually occur before photodetection will also produce out-of-band crosstalk. It might happen that crosstalk power from the neighboring channel “brings back” a part of power that originally belonged to the channel in question. Although that portion contains the same data as channel in question, they are not in phase with each other any more since they experienced different delays before reunion occurred. The same process (separation and reunion) might happen in the optical switch due to nonideal isolation between switch ports, as shown in Figure 4.5b.

In-band crosstalk will eventually produce the beat noise components, similar to the case when the ASE noise from optical amplifiers was involved. The only real difference is that the crosstalk, rather than the ASE noise, will beat with the signal and with itself. The total beat photocurrent in the $m$-th channel is

$$i_{cross,m}(t) = \Re\left[E_m(t)\exp[-j\omega_m t + \phi_m(t)]\right] + \sum_{n=1}^{m-1} X_n E_n(t) \exp[-j\omega_n t + \phi_n(t)]$$  \hspace{1cm} (4.46)
where $R$ is photodiode responsivity, $E_m$ is the electric field of the optical signal in question, while $X_n$ and $\phi_n$ are the amplitudes and phases the electric fields associated with in-band crosstalk components. The parameter $M$ denotes the potential number of in-band contributions. Please notice that the exponential term denotes the coherent nature of the total electric field. The Equation (4.46) can be sorted out by using substitution $E = \sqrt{P}$ and by performing all multiplications, so the total current becomes

$$i_{\text{cross,inc}}(t) = RP_m + 2R \sum_{n=1}^{M} \sqrt{P_m} \cos[\phi_m(t) - \phi_n(t)] = R(P_m + \Delta P_{\text{cross,inc}}) \quad (4.47)$$

As we can see from Equation (4.47) that in-band crosstalk has the same nature as the intensity noise and can be treated as a component of the total the intensity noise.

4.2 DEFINITION OF BER, SNR, AND RECEIVER SENSITIVITY

Optical signal that is distorted from the original shape during its travel along a lightwave path is eventually converted to the photocurrent by photodiode in an optical receiver. All acquired impairments will also be transferred to the electrical form. In addition, new noise components as corruptive additives will be generated in optical receiver, as discussed above. Both the distorted signal and corruptive additives will eventually come together to the decision circuit that recognizes logical levels at defined clock intervals, as shown in Figure 4.2. The signal value at the decision point should be as high as possible to keep a favorable distance from the noise level, and to possibly compensate for the receiver sensitivity degradations due to impact of other various impairments (such as dispersion, a finite extinction ratio, crosstalk, etc.).

Signal distortion due to impact of some impairments is illustrated in Figure 4.6a. In general, distortions are observed through pulse level decrease, pulse shape distortion, phase and frequency shift, or as the noise additives. The receiver sensitivity degradation due to pulse level decrease can be evaluated directly, while the evaluation of the pulse shape distortion or pulse phase change have more complex character. On the other hand, the impact of the noise is evaluated through the averaged power of the stochastic process.

Before we start discussion, we should mention that the majority of the topics covered in following sections are related to optical transmission systems with intensity modulation (IM) and direct detection (DD). Accordingly, all discussion serves as a foundation of more advanced topics discussed in Chapters 5, 6, and 7.
In this section we will first evaluate signal to noise ratio (SNR) and receiver sensitivity since both of them define the transmission quality. In next section, the impacts of other impairments will be quantified through the receiver sensitivity degradation with respect to a reference case. In some cases the receiver sensitivity degradation can be considered as the signal power penalty that needs to be compensated for. It can be done by allocating some power margin, which is defined as increase in the signal power that is needed to compensate for the impact of each specific impairment. By increasing the signal power, the SNR is kept at the same level that would exist in case where no impairment was involved. The signal increase to compensate for the power penalty could have a real value in some situations, such as those related to the impacts of chromatic dispersion or smaller extinction ratio. An increase in signal power would certainly compensate for the penalty associated with the impairment in question, unless a negative impact of nonlinear effects eliminates all benefits that would be brought with the signal power increase. On the other hand, the power margin has a different meaning if it is related to the impact of nonlinear effects. That is because increase in signal power would not be beneficial since it would also contribute to signal distortion. Regardless on that, the power penalty serves as a design parameter for evaluation of the nonlinear impairments, and for assessment and optimization of overall transmission capabilities.

### 4.2.1 Bit Error Rate and Signal to Noise Rate for IM/DD Scheme

The received SNR and the bit-error-rate (BER) at the output of the optical receivers are the parameters most commonly used to define transmission quality. In this section we will evaluate these parameters for the fundamental detection scenario where the stream of intensity modulated binary symbols, which occupy the entire slots (i.e., non return to zero- NRZ signals), is converted to an electrical signal by using direct detection scheme. The other schemes involving advanced
modulation formats and/or coherent detection will be analyzed in Chapters 5 and 6.

The SNR defines the difference between signal and noise levels at the sampling points. The BER is interrelated with SNR and defines the probability that a digital signal space (or “0” bits) will be mistaken for a digital signal mark (or “1” bits), and vice versa. The fluctuating signal levels that correspond to “1” or “0” bits can be characterized by corresponding probability density functions, as shown in Figure 4.7. These levels fluctuate around their average values \( I_1 \) and \( I_0 \), which are associated with “1” and “0” bits, respectively.

![Figure 4.7 Probability density functions related to “1” and “0” bits](image)

These two currents can be expressed as \( I_0 = R P_0 \) and \( I_1 = R P_1 \), where \( P_0 \) is the optical power during "0" bit, \( P_1 \) is the incoming optical power during "1", while \( R \) is the responsivity of the photodiode. Any current fluctuations around the average value are associated with the noise. The noise intensity can be characterized by standard deviations \( \sigma_1 \) and \( \sigma_0 \), which are related to "1" and "0" bits, respectively. At the same time, the noise power associated with “1” and “0” bits can be characterized by variances \( (\sigma_1)^2 \) and \( (\sigma_0)^2 \).

The current levels at the decision circuit are sampled at moments corresponding to the recovered signal clock, and then compared with some threshold value \( I_{th} \). Bit “1” is recovered if the sampled value is higher than the threshold, while “0” bit is recovered if the sampled value is lower than the threshold. The decision is correct if coincides with the situation at the transmitting side. However, an error occurs if “1” bit is recovered when “0” bit was sent. The reason for such error lies in fact that fluctuating current at the decision instant was high enough to cross the threshold value, and was recognized as the level associated with “1” bit. Another false decision occurs if bit “0” is recovered when bit “1” was sent. Such decision has been done since current fluctuation around average value \( I_1 \) was relatively high in the negative direction. It eventually went below the threshold level, and when compared to threshold, a false decision was made.

The bit error rate (BER) accounts for both cases of false decision. Accordingly, the total probability of a false decision can be expressed as
BER = \( p(1)P(0/1) + p(0)P(1/0) = 0.5[P(0/1) + P(1/0)] \) \( (4.48) \)

where \( p(0) \) and \( p(1) \) are probabilities that 0 and 1 were transmitted. We can safely assume that \( p(0) = p(1) = 0.5 \), which applies for a longer data bit stream. The conditional probabilities \( P(0/1) \) and \( P(1/0) \) are respectively related to cases when “0” was recovered while “1” was sent, and when “1” was recovered while “0” was sent. Probability \( P(0/1) \) is represented by the area under \( P(1) \) function that is placed below the threshold level, as shown in Figure 4.7. At the same time, the probability \( P(1/0) \) can be identified as the area under function \( P(0) \) that lies above the threshold line.

The calculation of BER parameter by Equation (4.48) involves both the signal and noise parameters. Signal is characterized by average values \( I_1 \) and \( I_0 \), while the total noise is characterized by standard deviations \( \sigma_1 \) and \( \sigma_0 \), which depend on the intensity of different noise components that might contribute to fluctuation of the total current. It is therefore

\[
\sigma_1 = \sqrt{\langle i_1^2 \rangle_{\text{total}}} \quad (4.49)
\]

\[
\sigma_0 = \sqrt{\langle i_0^2 \rangle_{\text{total}}} \quad (4.50)
\]

where \( i_{1,\text{total}} \) and \( i_{0,\text{total}} \) are the fluctuating currents that are related to “1” and “0” bits, respectively. The statistics of the current fluctuations at the sampling points is rather complex, and an exact calculation of BER is rather tedious. However, there are several fairly good approximations that are used so far to evaluate the BER in optical receivers [20], [21]. The simplest, yet effective method is based on assumption that both probability functions related to noise are the Gaussian distributions, which are characterized by the mean and standard deviation—please refer to Figure 4.7.

The Gaussian model for noise functions leads to the following expressions for conditional probabilities \( P(0/1) \) and \( P(1/0) \)

\[
P(0/1) = \frac{1}{\sigma_1 \sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left[ -\frac{(I - I_1)^2}{2\sigma_1^2} \right] dl = \frac{1}{2} \text{erfc} \left( \frac{I_1 - I_{\text{th}}}{\sigma_1 \sqrt{2}} \right) \quad (4.51)
\]

\[
P(1/0) = \frac{1}{\sigma_0 \sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left[ -\frac{(I - I_0)^2}{2\sigma_0^2} \right] dl = \frac{1}{2} \text{erfc} \left( \frac{I_0 - I_{\text{th}}}{\sigma_0 \sqrt{2}} \right) \quad (4.52)
\]

where \( \text{erfc}(x) \) is complementary error function, which is defined as [1], [22]
\[ \text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-y^2} dy \] (4.53)

Both probabilities from Equations (4.51) and (4.52) depend on the threshold value \( I_{th} \), which means that the threshold value can be adjusted in order to reduce the probabilities of false detection. The threshold adjustment can be done by equalizing arguments in Equations (4.51-4.52), which leads to

\[
\frac{(I_i - I_{th})^2}{2\sigma_i^2} = \frac{(I_o - I_{th})^2}{2\sigma_o^2} + \ln \left( \frac{\sigma_i}{\sigma_o} \right) = \frac{(I_i - I_o)^2}{2\sigma_o^2}
\] (4.54)

The optimum threshold value obtained from Equation (4.54) is

\[
I_{th} = \frac{\sigma_i I_o + \sigma_o I_i}{\sigma_i + \sigma_o}
\] (4.55)

which is a valid approximation when \( \sigma_i \) is close to \( \sigma_o \). The above value for \( I_{th} \) can be inserted in Equations (4.51-4.52); the values \( P(0/1) \) and \( P(1/0) \) can be inserted back to Equation (4.48), so it becomes

\[
\text{BER} = \text{BER}(Q) = \frac{\text{erfc}(Q/\sqrt{2})}{2} \approx \frac{\exp(Q^2/2)}{Q\sqrt{2\pi}}
\] (4.56)

The approximate expression on the right side of above equation is reasonably accurate for \( Q > 4 \).

The \( Q \)-factor from Equation (4.56) is defined as

\[
Q = \frac{I_i - I_o}{\sigma_i + \sigma_o}
\] (4.57)

This parameter is often taken as a direct measure of the SNR, although the exact relationship between \( Q \)-factor and SNR depends on the detection scheme that is used. The \( Q \)-factor can be evaluated experimentally through so-called “eye diagram” presented at the oscilloscope screen. The eye diagram is obtained by superposition of several sequences of received signal form on top of each other. Each sequence is usually several bits long. The eye diagram should be as open as possible and as clear as possible, and it can be very useful to estimate the impact of different impairments. Functional dependence \( \text{BER}(Q) \) is shown in Figure 4.8, and serves as one of the most important tools that are used in system performance evaluation. This function returns several reference points, shown in Figure 4.8, that can be used in many practical considerations, and they are:

- \( \text{BER} = 10^{-9} \) which corresponds to \( Q = 6 \), or \( 20 \log(6) = 15.65 \) dB
- \( \text{BER} = 10^{-12} \) which corresponds to \( Q = 7 \), or \( 20 \log(7) = 16.90 \) dB
- $BER = 10^{-15}$ which corresponds to $Q = 8$, or $20 \log(8) = 18.06$ dB

The factor 20, rather than 10 was used to calculate decibels value. It is because $Q$-factor is related to the electrical level, and $20 \cdot \log(Q)$ measures the power levels. It should be also mentioned that $Q^2$ is sometimes used in association with the optical domain, which is based on the fact that the value $20 \cdot \log(Q)$ is equal to $10 \cdot \log(Q^2)$.

**Figure 4.8 BER as a function of $Q$-factor**

### 4.2.2 Optical Receiver Sensitivity

The performance of the optical receiver can be evaluated by the parameter $P_R$, known as the receiver sensitivity. The receiver sensitivity is defined as the average power needed to achieve $BER$ that is lower or equal to a specified value, i.e.

$$P_R = \frac{P_0 + P_1}{2} = \frac{N_{\text{photon}} h \nu}{2T} = \langle N_{\text{photon}} \rangle B h \nu$$  \hspace{1cm} (4.58)

where $P_1$ and $P_0$ are power levels related to “1” and “0” bits, respectively, $T$ is the bit duration, $B$ is signal bit rate, $N_{\text{photon}}$ is the average number of photons carried by each “1” bit, $\langle N_{\text{photon}} \rangle = N_{\text{photon}} / 2$ is the average number of photons extended over streams of “1” and “0” bits.

Receiver sensitivity is very often specified with respect to the three values of $BER$ written above. A minimum number of photons needed to achieve specified $BER$ could be evaluated through Equation (4.18), which can be rewritten as

$$P(n) = \frac{N_{\text{photon}}^n e^{-N_{\text{photon}}}}{n!}$$  \hspace{1cm} (4.59)
Above equation gives the probability of generating $n$ electron-hole pairs for the average number $N_{\text{photons}}$ of photons carried by each “1” bit. The BER of an ideal optical receiver can be calculated by using Equations (4.48) and (4.59). The probability $P(1/0)$ will be zero, since it is not possible to generate any electrons if there is no incoming photons. On the other hand, there is some probability that zero level will be recognized even if there are incoming photons. That probability can be obtained from Equation (5.49) for $n = 0$. Accordingly, we have that

$$P_R = \frac{P_0 + P_1}{2} = \frac{N_{\text{photons}}h\nu}{2T} = \langle N_{\text{photons}} \rangle Bh\nu$$

(4.60)

We can establish some initial reference points with respect to the receiver sensitivity by evaluating the sensitivity of an ideal optical receiver for specified bit rates. It is done below in three examples by applying Equations (4.58-4.60) to the exemplary high-speed bit rates at the wavelength of 1,550 nm.

**Example 1:** for $BER=10^{-9}$, $N_{\text{photons}} = 20$ or $\langle N_{\text{photons}} \rangle = 10$ is needed, which translates to the following sensitivities of ideal receivers:
- $P_R = -54.94$ dBm for signal bit rate $B = 1/T = 2.5$ Gb/s
- $P_R = -48.92$ dBm for signal bit rate $B = 10$ Gb/s
- $P_R = -42.90$ dBm for signal bit rate $B = 40$ Gb/s
- $P_R = -38.92$ dBm for signal bit rate $B = 100$ Gb/s

**Example 2:** for $BER=10^{-12}$, $N_{\text{photons}} = 27$ or $\langle N_{\text{photons}} \rangle = 14$ is needed, which translates to the following receiver sensitivities:
- $P_R = -53.64$ dBm for signal bit rate $B = 2.5$ Gb/s
- $P_R = -47.62$ dBm for signal bit rate $B = 10$ Gb/s
- $P_R = -41.59$ dBm for signal bit rate $B = 40$ Gb/s
- $P_R = -37.61$ dBm for signal bit rate $B = 100$ Gb/s

**Example 3:** for $BER=10^{-15}$, $N_{\text{photons}} = 34$ or $\langle N_{\text{photons}} \rangle = 17$ is needed, which translates to the following receiver sensitivities:
- $P_R = -52.63$ dBm for signal bit rate $B = 2.5$ Gb/s
- $P_R = -46.61$ dBm for signal bit rate $B = 10$ Gb/s
- $P_R = -40.59$ dBm for signal bit rate $B = 40$ Gb/s
- $P_R = -36.61$ dBm for signal bit rate $B = 100$ Gb/s

As we see, the number of photons that is needed to achieve specified $BER$ could be acquired easier for longer bit intervals. That is the reason why receiver sensitivity is better for lower bit rates (better receiver sensitivity is associated with lower $P_R$).

Any specific practical case can be characterized with associated receiver sensitivity, which will be different than one related to an ideal optical receiver. That is because the sensitivity of an ideal optical receiver is determined only by the quantum limit of photodetection, while the impact of any other signal impairments is not included. However, these impacts should be included while considering any practical case. Different impairments will degrade receiver...
sensitivity, which is observed through increase in power $P_R$ that is required to keep specified BER. The difference between the receiver sensitivities related to an ideal optical receiver and to the practical application scenario can be considered as the optical power penalty associated to that specific case. The biggest contribution to the overall receiver sensitivity degradation comes from the noise side. Since the noise accompanies signal in all practical cases, it is convenient to evaluate the receiver sensitivity degradation due to noise impact and establish new reference points that are more relevant from design perspective than the ones related to the ideal receiver case.

4.2.2.1 Receiver Sensitivity Defined by Shot Noise and Thermal Noise

The impacts of the thermal and quantum shot noise components should be evaluated first since they are present in any photodetection scenario. It can be done by considering a direct detection scheme with no optical preamplification involved. We can assume that signal has an indefinite extinction ratio, which means that the power $P_0$ carried by “0” bits can be neglected. In such a case, the receiver sensitivity from Equation (4.58) becomes

$$P_R = \frac{P_1}{2} = \frac{RI_1}{2}$$

(4.61)

where $R$ is photodiode responsivity given in A/W.

We can use values from Table 4.2 to recognize the noise components that are dominant in different detection scenarios including this one. Some other noise components, including the dark current noise, can be neglected since they have smaller contribution. Accordingly, one can recognize that the thermal noise dominates for “0” bits in direct detection scenario, while both the thermal noise and quantum shot noise contribute to the total noise for “1” bits. The parameters that define $Q$-factor in this detection scenario will have the following values

$$I_1 = R\langle M \rangle P_1 = 2\langle M \rangle RP_R$$

(4.62)

$$\sigma_i^2 = \langle \hat{i}_i^2 \rangle_{total} = \langle \hat{i}_i^2 \rangle_{same} + \langle \hat{i}_i^2 \rangle_{shot} = 2q\langle M \rangle^2 F(M) I_1 \Delta f + \frac{4k\Theta NF_{\alpha} \Delta f}{R_L}$$

(4.63)

$$\sigma_0^2 = \langle \hat{i}_0^2 \rangle_{total} = \langle \hat{i}_0^2 \rangle_{shot} = \frac{4k\Theta NF_{\alpha} \Delta f}{R_L}$$

(4.64)

where $\langle M \rangle$ is the average value of photodiode amplification parameter, while $F(M)$ is photodiode noise factor. The amplification parameter is always larger than 1 if APD is used, while it equals 1 if the PIN photodiode is employed. The noise factor $F(M)$ has unity value if it is related to the PIN photodiodes, while it higher if it is related to APD and can be evaluated by using Equations (4.27) and (4.28).

The $Q$-factor in this case can be expressed as
\[ Q = \frac{I}{\sigma_1 + \sigma_0} = \frac{2(M)R_P}{\left[ 2g(M)^2 F(M) 2(M)R_P \Delta f + \frac{4k\Theta NF_{ne} \Delta f}{R_L} \right] + \frac{4k\Theta NF_{ne} \Delta f}{R_L}} \]  

(4.65)

It is possible to solve above equation with respect to \( P_R \), and it becomes [14]

\[ P_R = \frac{Q \left( \frac{1}{\langle M \rangle_{\text{th}}^2} \right)}{\langle M \rangle R} + \frac{q \Theta^2 F(M) \Delta f}{R} = \frac{1}{\langle M \rangle} \left[ \frac{Q(4k\Theta \cdot NF_{ne} \Delta f)^{1/2}}{R \sqrt{R_L}} \right] + \frac{q \Theta^2 F(M) \Delta f}{R} \]  

(4.66)

The thermal noise term dominates in Equation (4.66) if PIN photodiode is used since \( \langle M \rangle = F(M) = 1 \), and this detection scenario is recognized as the thermal-noise limited case [14], [16], [17]. The receiver sensitivity in thermal-noise limited case can be calculated by neglecting the shot noise contribution in Equation (4.66), which leads to

\[ P_{R,\text{PIN}} = \frac{\sigma_{\text{th}} Q}{R} \frac{Q(4k\Theta \cdot NF_{ne} \Delta f)^{1/2}}{R \sqrt{R_L}} \]  

(4.67)

Therefore, the receiver sensitivity is determined by the receiver bandwidth \( \Delta f \), the load resistor \( R_L \), and the noise figure of the front-end amplifier \( NF_{ne} \). We can compare the thermal noise limited case with an ideal optical receiver case by assuming that \( \Delta f = B \), although the receiver bandwidth depends on the modulation format and can range anywhere from \( 0.5B \) to \( B \).

![Optical receiver sensitivity](image.png)

**Figure 4.9** Optical receiver sensitivity
Receiver sensitivity for thermal-noise limited case and sensitivity degradation from the value associated with an ideal receiver is shown in Figure 4.9 for several bit rates of practical interest (i.e. for \( B = 0.1, 1, 2.5, 10, 40, \) and 40 Gb/s), and for the \( BER = 10^{-12} \). The sensitivity that would apply if there were direct detection of signals with bit rate of 100 Gb/s is also denoted in Figure 4.9 just for comparison sake. It was assumed that \( R = 0.8 \) A/W and \( R_L = 50 \) \( \Omega \). Please notice that sensitivity degradation is smaller for higher bit rates. That is because in such a case the noise power increases in proportion to square root of the signal bandwidth (bit rate), rather than in proportion to the bit rate.

The SNR in the thermal-noise limited case can be calculated as

\[
SNR = \frac{I^2}{\sigma_n^2} = 4Q^2
\] (4.68)

Accordingly, the following SNR values can be associated with Q-factors:
- \( SNR = 144 \) or 21.58 dB for \( Q = 6 \), which provides \( BER = 10^{-9} \)
- \( SNR = 196 \) or 22.92 dB for \( Q = 7 \), which provides \( BER = 10^{-12} \)
- \( SNR = 256 \) or 24.08 dB for \( Q = 8 \), which provides \( BER = 10^{-15} \)

If APD are used, the avalanche amplification will enhance the signal by factor \(<M>\). At the same time, the additional shot noise, which is proportional to noise factor \( F(M) \), will be generated. In this case the shot noise contribution may become comparable or even larger than the thermal noise contribution. It is possible to find an optimum amplification factor \(<M>_{opt}\) that will optimize the receiver sensitivity, and to minimize the function \( P_{R}(<M>) \) given by Equation (4.66). For that purpose it is useful to employ Equation (4.27) for \( F(M) \). The optimization procedure applied to Equation (4.66) leads to the following value for the APD receiver sensitivity [6], [14]

\[
P_{R,APD} = \frac{qQ^2(2k_N<M>_{opt} + 2(1 - k_N))\Delta f}{R}
\] (4.69)

where an optimum value of the gain coefficient is given as

\[
<M>_{opt} = \left( \frac{\sqrt{i^2}}{k_N qQ\Delta f} - \frac{k_N - 1}{k_N} \right)^{1/2} \approx \left( \frac{\sqrt{i^2}}{k_N qQ\Delta f} \right)^{1/2}
\] (4.70)

We can use the noise parameters from Table 4.2 to evaluate sensitivity of the APD optical receivers. It is easy to show that the APD receiver sensitivity is typically enhanced by at least 5-10 dB as compared to sensitivity of the PIN based optical receivers. This benefit is limited to bit rates up to 10 Gb/s since the APD frequency range prevents them to be deployed in optical receivers operating efficiently at bit rates higher than 10 Gb/s. The receiver sensitivity for APD based
optical receivers with an optimum avalanche gain is shown in Figure 5.4. The application area is limited to 10 Gb/s, and that is the reason why a dash line is drawn in Figure 4.9 since it serves just for comparison purposes.

4.2.2.2 Receiver Sensitivity Defined by Optical Preamplifier

Receiver sensitivity of IM/DD scheme can be considerably enhanced in case when an optical amplifier is employed in front of photodiode. This method, also known as the optical preamplification, is the most efficient if it is used in combination with PIN photodiodes, since combination with APD could introduce relatively high shot noise and diminish the benefit of preamplification. However, if APD is used in combination with optical preamplifiers, the amplification coefficient \( <M> \) should be adjusted to relatively low values to suppress the impact of the shot noise. For instance, it might be a good choice to keep \( <M> \) to be lower than 5.

Receiver sensitivity can be evaluated by using the same approach that was used in previous part. We can again assume that there is an indefinite extinction ratio, which means that the power \( P_0 \) carried by “0” bits can be neglected. In addition, Table 4.2 can be used to estimate the significance of the individual noise components involved in this case. As we see, the beat noise components will be the strongest ones. This applies even if APD is used since the avalanche gain should be adjusted to lower values.

For this detection scenario we have again that \( P_0 = P/2 \), while the noise parameters can be obtained from Equations (4.42-4.43). Therefore, it is

\[
I_1 = RGP_R = 2GP_R
\]  

(4.71)

\[
\sigma_1^t = \langle i^2 \rangle_{\text{total}} = \langle i^2 \rangle_{\text{sig-sp}} + \langle i^2 \rangle_{\text{sp-sp}} = 8R^2GP_RS_{sp}\Delta f + 2R^2S_{sp}[2B_{sp} - \Delta f]\Delta f
\]  

\[
\sigma_0^t = \langle i^2 \rangle_{\text{total}} = \langle i^2 \rangle_{\text{sp-sp}} = 2R^2S_{sp}[2B_{sp} - \Delta f]\Delta f
\]  

(4.72)

\[
Q = \frac{I_1}{\sigma_1 + \sigma_0} = \frac{2GP_R}{\langle i^2 \rangle_{\text{sig-sp}} + \langle i^2 \rangle_{\text{sp-sp}}^{1/2} + \langle i^2 \rangle_{\text{sp-sp}}^{1/2}}
\]  

(4.73)

Please notice that noise related to “0” bits is determined just by spontaneous-spontaneous beat component, since we neglected the power \( P_0 \) carried by “0” bits.

The receiver sensitivity extracted from above equations is

\[
P_{R} = \frac{2S_{sp}\Delta f}{G-1} \left[ Q + Q \left[ \frac{B_{sp} - \Delta f}{\Delta f} - 0.5 \right]^{1/2} \right] = NF_{sp}h\beta\Delta f \left[ Q + Q \left[ \frac{B_{sp} - \Delta f}{\Delta f} - 0.5 \right]^{1/2} \right]
\]  

(4.74)

In above equation, we used Equation (4.34) between spontaneous noise power density and the optical amplifier noise figure to express \( S_{sp} \) as
We can conclude from Equation (4.75) that optical amplifiers with low noise figure should be used in order to fully utilize benefits brought by optical preamplification. In addition, it is necessary to adjust the optical filter bandwidth to be as close as possible to the signal bandwidth to minimize the noise impact.

The receiver sensitivity related to the receiver with an optical preamplifier were again calculated for five exemplary bit rates, and for \( BER=10^{-12} \). All results were obtained by assuming that optical filter has a bandwidth \( B_{op}=2\Delta f \). Obtained results are shown in Figure 4.9 in parallel with results obtained for other cases. The results from this figure prove that optical amplification is very beneficial since receiver sensitivity associated to this detection scheme is the one closest to sensitivity of an ideal optical receiver.

4.2.2.3 Receiver Sensitivity in Coherent Detection Schemes

The analysis applied so far for calculation of \( Q \)-factor and \( BER \) in optical receivers with a direct detection cannot be applied for optical receivers with coherent detection. These receivers will be analyzed separately in Chapter 6. The main reason for that is in fact that signal and noise statistics will be dependent on the modulation scheme that is applied. However, it is possible to estimate the SNR value and receiver sensitivity for a generic case of a coherent detection.

Coherent receiver is characterized by presence of a local laser oscillator, as shown in Figure 4.2. Contribution of the local laser oscillator dominate in creation of major noise components, which are: quantum shot noise, noise due to beating between electrical fields related to ASE and to local laser oscillator, and the laser intensity and phase noise. Coherent detection can be heterodyne or homodyne in nature. In first case, there is some difference between carrier frequencies of the incoming optical signal and local oscillator, while in homodyne detection scenario the frequency of the local optical oscillator is equal to the frequency of incoming optical signal. In general case, the photocurrent generated by the total optical power coming to photodiode can be presented as \[8\]

\[
I(t) = R P(t) = R \left[ P_{LO} + 2 \sqrt{P_s(t) P_{LO}} \cos \left( \omega_s - \omega_{LO} \right) + \Delta \phi(t) + \Delta \phi_s(t) \right] \quad (4.77)
\]

where \( P_s \) and \( P_{LO} \) are optical powers of the incoming signal and local laser oscillator, while \( \omega_s \) and \( \omega_{LO} \) are carrier frequencies of the incoming signal and local oscillator, respectively. Parameter \( \Delta \phi \) is phase difference between incoming signal and local oscillator, while and \( \Delta \phi_s(t) \) presents phase noise component described by Equations (4.9) and (4.10). Since the homodyne detection is just a special case of heterodyne detection (where \( \omega_s = \omega_{LO} \)) we will analyze just a
In heterodyne detection scheme, the power of the shot noise will be enhanced due to contribution of local oscillator power. The total variance of the shot noise component will depend on the binary state that was sent if ASK modulation schemes are applied, while it irrelevant for FSK and PSK modulation schemes. The total bandwidth \( \Delta f \) of the optical receiver in heterodyne detection scheme is equal to doubled value of the receiver bandwidth in direction detection scheme, while receiver bandwidth in homodyne detection case is equal to the bandwidth applied in direction detection scheme. However, in advanced modulation schemes with balanced optical receiver and M-ary modulation formats, the bandwidth of the optical receiver for both heterodyne and homodyne detection become the same and equal to \( \Delta f \), since frequency downshift filtering is applied in heterodyne detection.

If we assume that binary PSK (BPSK) modulation format is applied, the signal and noise parameters associated with the heterodyne detection can be expressed as

\[
I_i = I_o = 2R|M| \left[ P_i(t)P_{LO} \right]^{1/2}
\]

\[
\sigma_i^2 = \left( \sum_{k=1}^{2} \langle i_k^2 \rangle_{total} \right) = \langle i_S^2 \rangle_{SN,S} + \langle i_{SN,LO}^2 \rangle + \langle i_{LO}^2 \rangle_{LOLO} + (RP_{LO}P_{int}^2)
\]

\[
\sigma_o^2 = \left( \sum_{k=1}^{2} \langle o_k^2 \rangle_{total} \right) = \langle o_S^2 \rangle_{SN,S} \]

where with term \( RP_{LO}P_{int}^2 = 2(RP_{LO}P_{LO})^2RIN_{laser} = 2(RP_{LO}P_{LO})^2RIN_{LO} \) we included the impact of the intensity noise in accordance with Equations (4.16) and (4.127). The major contribution to the intensity noise is coming from local laser oscillator. It was again assumed that photodiode produces some gain \( <M> \), while photodiode responsivity is expressed through parameter \( R \). We should outline that PIN photodiode \( <M> = 1 \) is the best fit for coherent detection schemes. The application of APD is limited to cases when gain is kept low \( <M> \leq [3 \rightarrow 5] \).

Components \( \langle i_S^2 \rangle_{SN,S} \) and \( \langle i_{SN,LO}^2 \rangle \), where \( \langle i_S^2 \rangle_{SN,S} \ll \langle i_{SN,LO}^2 \rangle \), are shot noise components created by signal optical power and by local laser oscillator, respectively, while \( \langle i_{LO}^2 \rangle_{LOLO} \) is the power of the thermal noise component. Component \( \langle i_{LO}^2 \rangle_{LOLO} \) presents the beating noise due interaction of electrical fields of ASE and local laser oscillator. This component resembles signal-spontaneous beat noise in scenarios with direct detection and can be evaluated by applying Equation (4.42) for this specific case, while replacing the product \( G \cdot P \) with \( P_{LO} \), while assuming that spontaneous emission is coming from optical preamplifier but from the remote, which means that its power is now equal \( S_{sp}/G \). We should also notice that spontaneous-spontaneous beat noise component is small and not relevant any more.

Also, the impact of the intensity noise of the local laser oscillator have to included, as it is done through power of the intensity noise currents given by last
terms in Equations (4.79) and (4.80). Parameter \( r_{int} \), defined by Equation (4.16), is applied to local laser oscillator, which power is now dominant. As for the impact of the intensity noise, please refer also to Equation (4.129) and Figure (4.18). We can assume that the impact caused by intensity noise of the local laser oscillator will be similar to the impact of the transmitter laser intensity noise, but much stronger in magnitude. We can also notice that contribution of the dark current has been neglected. Accordingly, the dominant noise components from Equations (4.79) and (4.80) are calculated as

\[
\langle i^2 \rangle_{sp, LO} = 4R^2P_{LO}S_{\text{sp}}/G = S_{sp, LO}M_{\text{equiv}}
\]

\[
\langle i^2 \rangle_{sh, LO} = 2g(M)^2 F(M)RP_{LO} \Delta f_{\text{equiv}}
\]

\[
\langle i^2 \rangle_{he} = \frac{4\Delta \Theta \cdot NF_{he} \Delta f_{\text{equiv}}}{R_L}
\]

where \( \Delta f_{\text{equiv}} \) presents the equivalent receiver bandwidth. In general case it is: (i) \( \Delta f_{\text{equiv}} = 2\Delta f \) for heterodyne detection, (ii) \( \Delta f_{\text{equiv}} = \Delta f \) for homodyne detection (\( \Delta f \) is receiver bandwidth that determines the noise bandwidth in direct detection receivers). However, in coherent receivers with M-ary modulation formats and balanced detection both bandwidths become equal to \( \Delta f_{\text{equiv}} \).

The SNR for heterodyne detection scheme (in this case for BPSK modulation format) can be found as a ratio of the signal power to the total noise power. By applying Equations (4.78-4.83) we have that

\[
\text{SNR} = \frac{\langle i^2 \rangle_{\text{sp}}}{\sigma_i^2} = \frac{\left[ 2R(M)\left\{P(\tau)P_{\text{LO}}\right\} \right]}{2g(M)^2 F(M)RP_{\text{LO}} + 4R^2P_{\text{LO}}S_{\text{sp}}/G + 2(RP_{\text{LO}})^2 RIN_{\text{LO}} + 4\Delta \Theta \cdot NF_{he} \Delta f_{\text{equiv}}}
\]

\[
(4.84)
\]

We can calculate and compare we compare the power spectral densities of different noise components from Equation (4.84), in the same way we did it for direct detection scenario. They are obtained for the following typical values of the optical amplifier, local laser oscillator, and PIN photodiode parameters: \( P_{\text{LO}} \sim 10 \) dBm, \( F_{\text{no}} \sim 3.2 \) (which is 5 dB), \( R = 0.8 \) A/W, \( RIN_{\text{LO}} \sim -165 \) dB/Hz), \( G = 20 \) dB. The spectral densities from Equations (4.41), (4.42) and (4.43) now become \( S_{\text{sp, LO}} \sim 1.05 \cdot 10^{-20} \) A\(^2\)/Hz, \( S_{he} \sim 2 \cdot 10^{-23} \) A\(^2\)/Hz, and \( S_{sh, LO} \sim 2.5 \cdot 10^{-21} \) A\(^2\)/Hz, and \( S_{int} \sim 2(RP_{\text{LO}})^2 RIN_{\text{LO}} \sim 0.4 \cdot 10^{-20} \) A\(^2\)/Hz, respectively. There are several conclusions with respect to noise components in coherent detection scenario: (i) the impact of the thermal noise is suppressed as long as the power of local laser oscillator is larger than \( ~0.7 \) mW (in such case shot noise due to impact of local oscillator is approximately ten times larger than the thermal noise); (ii) the impact of beating noise caused by interaction of spontaneous emission and electric field of local
laser oscillator is dominant as long as the power of local oscillator is much higher than the power of incoming optical signal, which is true in almost all situations; (iii) the laser intensity noise is a major factor if $RIN_{LO}$ is higher than $-160$ dB/Hz (in which case that impact becomes comparable with the impact of the shot noise (however in balanced coherent receivers this impact is suppressed by at least 15 dB and becomes comparable with the impact of thermal noise). As a conclusion, we can say that coherent detection presents spontaneous emission dominant detection scenario with spectral power density determined by spectral power density of ASE noise, given by Equation (4.76), just enhanced proportionally to the power of local laser oscillator. In such a case, and for $<M> = 1$ we have the following approximation

$$SNR \approx \frac{4R^2\langle P_i \rangle P_{LO}G}{4R^2P_{LO}S_nN_{equiv}} = \frac{2\langle P_i \rangle G}{NF_nhv(G-1)\cdot \Delta f_{equiv}}$$

where $NF_n$ and $G$ are noise figure and gain of optical amplifiers producing ASE. Since coherent detection brings signal gain by mixing with the local oscillator, it is better use PIN photodiode to minimize the total noise (if APD is considered, that gain $<M>$ should be less than 3-5. Also, employment of an optical preamplifier in combination with heterodyne detection would not serve the purpose due to large noise, and is not considered for practical applications. Since in most coherent detection schemes optical power per bit is equal for both “1” and “0” bits, we can assume that receiver sensitivity $P_R$ is equal $<P_S>$, which is then

$$P_R \approx \frac{NF_nhv(G-1)\cdot \Delta f_{equiv}SNR}{2G}$$

As we already mentioned, the error probability in coherent selection schemes is modulation format specific and should be separately calculated for any case in question, where the impact of the laser phase noise should be also accounted for. As an example, the error probability in BPSK modulation scheme can be calculated as [8]

$$BER_{BPSK} = BER(SNR_{BPSK}) = \frac{1}{2} \int_{-\infty}^{\infty} p(\Delta \phi_n) \text{erfc} \left[ \frac{SNR_{BPSK} \cos \Delta \phi_n}{\sqrt{2}} \right] d(\Delta \phi_n)$$

where $p(\Delta \phi_n)$ is the probability distribution function of the random noise phase. It is often assumed that probability distribution function can be approximated by Gaussian distribution with variance given by Equation (4.10), i.e.

$$\langle \Delta \phi_n^2 \rangle = 4 \int_0^\infty S_\nu(\nu) \sin^2(\frac{\nu \Delta \phi_n}{2}) d\nu$$

(4.88)
where \( S_f(\nu) \) is the spectral density function of the frequency noise defined by Equation (4.8). The most relevant advanced modulation and detection schemes will be analyzed in Chapter 5 and Chapter 6.

### 4.2.3 Optical Signal to Noise Ratio

Optical Signal to Noise Ratio (OSNR) is very important parameter in monitoring the optical transmission quality. As its name says, the optical noise power is compared with the optical signal power while both of them are still in the optical domain. The main reason for OSNR evaluation is optical amplification process of the signal and all side-effects of that process. Namely, optical amplification process is accompanied by generation of amplified spontaneous emission (ASE) noise that accumulates along the transmission line. The power of the ASE noise is calculated with respect to a specified optical bandwidth \( B_{op} \). It can also be measured, which is usually done by optical spectrum analyzers (OSA) that evaluate the power of the ASE noise in assigned optical bandwidth slot. It is often required to calculate and measure the ratio of the optical signal power and the ASE noise at any specific point along the lightwave path. The OSNT serves as the main constraint in impairment aware optical routing that will be discussed in Chapter 8.

The optical signal to noise ratio at some point along the lightwave path can be defined

\[
\text{OSNR} = \frac{P_S}{P_{\text{ASE}}} = \frac{P_S}{S_{op}B_{op}} = \frac{P_S}{2n_{sp}h\nu(G-1)B_{op}}
\]

where \( P_S \) is the optical signal power at that specific point, \( n_{sp} \) is the spontaneous emission factor, \( G \) is optical amplifier gain, \( B_{op} \) is bandwidth of the optical filter, \( h \) is the Planck’s constant, while \( \nu \) is frequency of the optical signal. The factor 2 in the denominator at the right side accounts for two polarization modes of ASE, where each of them carries optical power equal to \( n_{sp}h\nu(G-1)B_{op} \). The optical filter bandwidth, which is equal to measure slot of the optical spectrum analyzer, is usually declared during the measurement process. For instance, there is a number of measurements in the optical domain that are done within the optical bandwidth equal 0.1 nm, which is approximately 12.5 GHz if applied to the 1,550 nm wavelength region.

The OSNR parameter can be also measured at the receiver entrance point just before photodetection takes place. In such a case, the optical power of the signal can be connected with the receiver sensitivity \( P_R \) by using relation \( P_S \sim 2P_R \), while the OSNR can be expressed by using Equation (4.75) and (4.89), i. e.
The OSNR value will be eventually converted to an electrical equivalent that defines the $Q$-factor and BER. The $Q$-factor in this detection scenario will be mainly determined by the ASE noise that is eventually converted to beat noise components. The following relationship between the $Q$-factor and OSNR can be established for NRZ signals if we neglect all other noise contributions except the beat noise components [23]

$$Q = \frac{2\text{OSNR} \sqrt{B_{op} / \Delta f}}{1 + \sqrt{1 + 4\text{OSNR}}}$$  \hspace{1cm} (4.91)$$

Equation (4.91) can be further simplified if we assume that there is just one dominant noise term, in this case signal-spontaneous beat noise from Equation (4.72). With this assumption Equation (4.91) becomes

$$Q \approx \frac{\sqrt{\text{OSNR} B_{op}}}{2 \Delta f}$$  \hspace{1cm} (4.92)$$

From transmission quality perspective, it is the $Q$-factor (which is proportional to SNR) that is the most important parameter since it is directly correlated with BER. Accordingly, from system design and performance monitoring perspective, it became necessary to have a good correlation between $Q$-factor and both SNR and OSNR in different detection scenarios. In some situations it can be done by using approximate empiric formulas, such as those given by Equations (4.68) and (4.92), but in many other scenarios it is useful to establish more precise relationship between $Q$-factor and OSNR. The relation between $Q$-factor and OSNR is transmission case specific, which means that it should be calculated for input parameters related to transmission scenario in question. For instance, Equations (4.66), (4.67), (4.69), (4.74) and (4.85) can be used to identify the receiver sensitivity for a specific performance requirement that is defined through BER, SNR, or $Q$-factor. That receiver sensitivity can be used afterwards as a reference to identify OSNR.

More precise correlation between $Q$-factor, SNR, and OSNR can be established by using numerical calculations, possibly with some measured data inputs. This process can be also performed on a dynamic base. Namely, OSNR can be measured along the lightwave path, while the values for $Q$ parameter and BER can be calculated for that specific scenario. The calculated values of the $Q$-factor can then be compared with established reference and used in the system performance monitoring process. This approach is obviously more sophisticated,
but it might be an inevitable part of any high-speed transmission system engineering, especially in an optical networking environment.

4.3 SIGNAL IMPAIRMENTS

Different linear and nonlinear effects discussed in Chapter 2 and Chapter 3 will cause optical signal distortion during its propagation along the lightpath. The impact of these signal impairments can be calculated either through the SNR by accounting for the overall signal change, or through optical power penalty related to the receiver sensitivity degradation. The impact of some impairments, such as fiber loss, is very straightforward and can be accounted directly through the SNR. On the other hand, the impact of most other impairments is more complex and needs to be evaluated through receiver sensitivity degradation. The precise analysis of the impact of various impairments is done by using numerical methods and finding solution of Nonlinear Schrodinger Equation. However, the purpose of analysis in this section is to evaluate the impact of impairments by using analytical tools, which will lead to closed form expressions. These expressions that are, in essence, approximate in nature, but accurate enough for practical system considerations, especially in field conditions.

If the optical power loss were the only cause of the signal distortion, incoming signal to the photodetector would be expressed as

\[ P_2(t) = P_1(t) - \alpha L - \alpha_c - \alpha_{\text{others}} \]  

(4.93)

where \( P_2(t) \) is the power of the signal that passed through different elements (fiber, splices, filters, couplers, switches, etc.), \( P_1(t) \) is the launched power at the beginning of the lightpath, \( \alpha_c \) are splicing and connector losses, while \( \alpha_{\text{others}} \) represent all other insertion losses that might occur along the lightpath. The power \( P(t) \) would directly determine the SNR, which means power loss translates directly to the power penalty.

The fiber loss coefficient is about 0.2 dB/km at wavelengths around 1,550 nm for single mode optical fibers based on doped silica. It can be even lower for so-called PSCF (pure silica core fibers), with the reported loss \( \alpha = 0.1484 \) dB/km [24]. The loss increases in value up to about 20% for other wavelengths covered by C and L wavelength bands. The attenuation in the S band is even higher and can be more than 50% above the attenuation value around 1,550 nm.

Typical mean attenuation inserted by fused optical splices is somewhere between 0.05 and 0.1 dB, while mechanical splices insert a loss comparable or slightly above of 0.1 dB. Optical connectors are designed to be removable, thus allowing many repeated connections and disconnections. There are several variants of single mode fiber optical connectors that are commonly used, such as FC, SC, LC, MU etc., all depending on the shape of the connector and mating.
sleeve shape [16]. The insertion loss for high quality single mode optical fibers should not be higher than 0.25 dB. A special connector design is very often applied to minimize reflection of the incoming optical signal from the surface of receiving part of the connector. Such a design can include angled fiber-end surfaces (FC/APC connector), or perhaps some index matching fluid applied at the fiber surfaces. The matching fluid will minimize the reflection coefficient given by Equation (4.17) by decreasing the refractive index differences when the optical signal crosses from one fiber to the other.

Optical connectors with angled fiber-end surfaces are the most convenient ones to minimize so-called “connector return loss”, which is a fraction of the optical power reflected back into the fiber at the connection point. In some cases, such as bi-directional transmission through the same fiber, the reflection power should be more than 60 dB below the input level. This requirement is satisfied by using connectors that provide a physical contact of the angled fiber ends.

The number of the optical splices and connectors depends on the length of lightwave path and should be taken into account during system design. Optical signal attenuation inserted due to fiber splicing and cabling is often distributed over the entire transmission length and added to the coefficient $\alpha$. Such a distributive approach is very useful in the system engineering process. It is common practice to add an additional fraction of about 10% to the coefficient $\alpha$ to account for the impact of fiber splicing and cabling. This is of course case when no measurements data are available.

It is clear, however, that Equation (4.93) captures only simplistic reference case. More realistic approach is when Equation (4.93) is expressed as

$$P_i(t) = P_i(t) - \alpha_L - \alpha_e - \alpha_{\text{others}} - \sum \Delta P_i(t)$$

(4.94)

where sum of $\Delta P_i$ accounts for receiver sensitivity degradation caused by signal impairments. In next part we will discuss the impact of the most relevant impairments to optical receiver sensitivity and evaluate $\Delta P_i$ for most relevant cases.

4.3.1 The Impact of Mode Dispersion in Multimode Fibers

The impact of mode dispersion in multimode fibers was evaluated by Equations (2.4) and (2.8) for step-index and graded-index refractive profile, respectively. It was also mentioned in Chapter 3 that the impact of the intermodal dispersion can be expressed through the fiber bandwidth given by Equation (3.78). The fiber bandwidth is a distance dependent parameter that can be calculated from the available fiber data for each specified transmission length. It is useful from the system perspective to convert the fiber bandwidth to “pulse broadening” parameter in multimode optical fibers, and take it as a part of a total system pulse spreading consideration as presented in Equations (4.178) and (4.179). The fiber
bandwidth can be converted to the pulse broadening time by using the following relation [25]

\[ \Delta t_{\text{fib},L} = \frac{U}{B_{\text{fib},L}} = \frac{UL^\mu}{B_{\text{fib}}} \]  \hspace{1cm} (4.95)

where \( L \) is the length of the fiber in question, \( B_{\text{fib}} \) is optical fiber bandwidth of a 1 kilometer optical fiber length, \( B_{\text{fib},L} \) is the bandwidth of the specified fiber length while \( \mu \) is coefficient which can take values that are in the range from 0.5 to 1. It is around \( \mu = 0.7 \) for most multimode optical fibers. Parameter \( U \) in Equation (4.95) represents the fact that the broadening time is also related to the modulation format. It is 0.35 for non-return to zero (NRZ) modulation formats, and 0.7 for return-to-zero (RZ) modulation formats. The broadening time for 1-km fiber length calculated by Equation (4.95) can vary anywhere from 0.5 ns (for graded-index fibers) to 100 ns (for step-index fibers).

The impact of chromatic dispersion on the system performance is a major factor of the pulse form degradation if transmission is done over single mode optical fibers. As for multimode fibers, it is generally smaller than the impact of intermodal dispersion. However, it can contribute significantly to the total pulse broadening if transmission is done far away from zero-dispersion region.

### 4.3.2 The Impact of Chromatic Dispersion

The total impact of chromatic dispersion in single mode optical fibers is considered in conjunction with several parameters that are related not just to characteristics of optical fiber, but also to the properties of the transmission signal, and to characteristics of the light source. The transmission system should be design to minimize the impact of chromatic dispersion. That impact can be evaluated through the signal power penalty, which in this case presents the signal power increase that is needed to keep the signal to noise ratio unchanged.

The impact of chromatic dispersion can be evaluated by assuming that the pulse spreading due to dispersion should not exceed a critical limit. That limit can be defined either by the fraction \( \delta_{\text{chrom}} \) of the signal that leaks outside of the bit period, or by the broadening ratio \( \delta_{\text{bchrom}} \) of widths associated to the input and output pulse shapes. These parameters do not translate directly to the power penalty since they deal with the pulse shape distortion, rather than to the amplitude decrease.

The exact evaluation of the power penalty could be rather complicated since it is related to a specific signal pulse shape. Instead, a reasonable approximate evaluation can be done by assuming that the pulse takes a Gaussian shape given by Equation (3.126), while using Equation (3.141) as a starting point. Herewith, we will adopt the approach of using broadening factor \( \delta_{\text{bchrom}} \) as a measure of the
The broadening factor can be expressed as

\[ \delta_{b,\text{chrom}} = \frac{\sigma_{\text{chrom}}}{\sigma_0(B)} \]  

(4.96)

where \( \sigma_{\text{chrom}} \) is the pulse root-mean square (RMS) width at the fiber end, while \( \sigma_0 \) is the pulse RMS at the fiber input. Please notice that \( \sigma_0 \) is expressed as a function of the signal bit rate \( B = 1/T \), where \( T \) defines the length of the bit interval. If the input has the Gaussian pulse shape, it will be confined within the time slot \( T \) if it satisfies the following relation

\[ \sigma_0 \leq \frac{T}{4} = \frac{1}{4B} \]  

(4.97)

where \( \sigma_0 \) is now related to RMS of the Gaussian pulse. In fact, it was shown in [17] that Equation (4.97) guarantees that almost 100% of the pulse energy is contained within the pulse interval. Any increase of the parameter \( \sigma_{\text{chrom}} \) above value \( \sigma_0 \), will indicate that there is some power penalty associated with that specific case. On the other hand, there is no penalty unless the RMS of the output pulse exceeds the RMS of the input pulse. The power penalty could be even negative if an initial pulse compression is observed.

We can estimate the power penalty \( \Delta P_{\text{chrom}} \) due to chromatic by using the following formula

\[ \Delta P_{\text{chrom}} = 10 \log (\delta_{b,\text{chrom}}) = 10 \log \left[ \frac{\sigma_{\text{chrom}}}{\sigma_0(B)} \right] \]  

(4.98)

The above formula is relatively simple, but the main task in calculating the power penalty \( \Delta P_{\text{chrom}} \) is related to calculation of the broadening factor \( \delta_{b,\text{chrom}} \). There are also some other more complex formulas that calculate the power penalty due to chromatic dispersion, such as one presented in [26]. The dispersion penalty given by Equation (4.98) can be calculated by applying Equation (3.141) to determine the broadening factor \( \delta_{b,\text{chrom}} \). It is often more appropriate to apply a specific equation from Table 3.2 for transmission scenario in question. In general case, the power penalty is a function of the initial pulse width, source linewidth, initial chirp parameter, chromatic dispersion parameter, and the transmission length.

**Example:** Let us consider the most common scenario when high-speed transmission is done out of zero-dispersion region, and when the light source spectrum is much smaller that the signal spectrum. We will calculate the pulse broadening parameter by using Equation (3.145). The results are obtained for the
following cases: \( \sigma_0 = 17.68 \text{ ps}, \ \sigma_0 = 7.06 \text{ ps}, \ \sigma_0 = 4.41 \text{ ps}, \ \sigma_0 = 1.77 \text{ ps} \), which corresponds to high-speed bit rates of 10 Gb/s, 25 Gb/s, 40 Gb/s, and 100 Gb/s, respectively. It is assumed that a single mode optical fiber with chromatic dispersion \( D = 17 \text{ ps)/(nm-km)} \) is used. The results are shown in Figures 4.10 and 4.11. It is also important to mention that bit rate can be translated to symbol rate if more advanced modulation format is used. As an example, if 100 Gb/s is modulated by applying QPSK format and polarization multiplex, the symbol rate will be 25 Gsymbol/s. We can see from Figures 4.10 and 4.11 that dispersion penalty is dependent of the initial value \( C_0 \) of the chirp parameter. It can be even positive, as mentioned above, since the pulse undergoes an initial compression. It occurs if negative initial chirp is combined with positive chromatic dispersion, and vice versa.

**Figure 4.10** Power penalties due to chromatic dispersion impact on 10 Gb/s and 25 Gb/s signal bit/symbol rates

**Figure 4.11** Power penalties due to chromatic dispersion impact on 40 Gb/s and 100 Gb/s signal bit/symbol rates
The real importance of the results from Figures 4.10 and 4.11 is that they show what amount of the dispersion can be tolerated for a specific dispersion penalty. The dispersion penalty limit can be established by defining a power penalty ceiling. It is commonly 0.5 dB or 1 dB, as shown by dotted lines in Figures 4.10 and 4.11. In some cases, such as for non-amplified point-to-point transmission with bit rates up to 10 Gb/s, even 2-dB power penalty can be tolerated. As an example, if \( C_0 = 0 \) for 10 Gb/s bit (symbol) rate and for 2-dB dispersion penalty, fiber length corresponding uncompensated chromatic dispersion will be will be \( \sim 38 \) km, which translates to dispersion of 608 ps/nm. At the same time, if \( C_0 = 0 \) for 40 Gb/s bit (symbol) rate and for 0.5-dB dispersion penalty, fiber length corresponding uncompensated chromatic dispersion will be \( \sim 1 \) km, which translates to chromatic dispersion of \( \sim 17 \) ps/nm.

The amount of chromatic dispersion that can be tolerated is also known as the “dispersion tolerance”. The dispersion tolerance with respect to 1 dB limit is summarized in Table 4.3 for the cases where no chirp is applied, and for different optical fibers in high bit rate transmission systems. The lengths of two different fiber types (standard SMF and NZDSF from Figure 3.8) that correspond to the dispersion tolerance are also shown in Table 4.3.

<table>
<thead>
<tr>
<th>Bit (symbol) rate in Gb/s</th>
<th>Dispersion in ps/nm</th>
<th>SMF Fiber length for ( D=17 ) ps/km-nm</th>
<th>NZDSF Fiber length for ( D=4 ) ps/km-nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>720</td>
<td>42</td>
<td>180</td>
</tr>
<tr>
<td>40</td>
<td>45</td>
<td>2.6</td>
<td>11</td>
</tr>
<tr>
<td>100</td>
<td>7.1</td>
<td>0.42</td>
<td>1.77</td>
</tr>
</tbody>
</table>

The dispersion penalty estimate can be simplified for two extreme cases, when optical source spectrum \( \sigma_\lambda \) is relatively wide, and when a chirpless Gaussian pulse propagates through the optical fiber. We can assume that the input Gaussian pulse has an optimum width \( 2\tau_0 = (\beta_2 L)^{1/2} \), where \( \beta_2 \) is the group velocity dispersion (GVD) parameter, while and \( 2\tau_0 \) represents the full width at 1/e intensity point (FWEM). These two extreme cases can be characterized by following equations derived from Equations (3.141) and (4.98)

\[
\sigma_\lambda L B \left( \frac{L}{2\pi} \right)^{1/2} < \delta_{s,\text{chrom}}, \quad \text{for the source with large spectral linewidth} \quad (4.99)
\]

\[
B \left( \frac{L}{2\pi} \right)^{1/2} < \delta_{s,\text{chrom}}, \quad \text{for an external modulation of CW laser} \quad (4.100)
\]

where \( B = 1/T \) is the signal bit rate, \( \sigma_\lambda \) is the source spectral linewidth, \( L \) is the transmission distance, while \( \delta_{s,\text{chrom}} \) is the fraction of the signal that leaks outside of the bit period \( T \). It is worth to mention that dispersion penalty limits are recommended in some early standard documents, such as [27], which states that the fraction spreading factor should be up to \( \delta_{s,\text{chrom}} = 0.306 \) for chromatic...
dispersion penalty below 1-dB, and lower than \( \delta_{s, \text{chrom}} = 0.491 \) for a 2-dB dispersion penalty.

The impact of chromatic dispersion in IM/DD systems with higher bit rates needs to be suppressed by proper selection of the parameters related to the pulse shape and optical modulator, and compensated by using proper compensation methods. In most cases chromatic dispersion have to be compensated and suppressed to a level that will cause a minimal power penalty (0.5 dB). The impact of chromatic dispersion, which is observed through the intersymbol interference (ISI), can be effectively canceled in coherent detection schemes by using digital filtering methods that require an intense signal procession at the electrical level—please refer to Chapter 6.

4.3.3 Polarization Mode Dispersion Impact

Polarization mode dispersion (PMD) effect discussed in Part 3.3.3 can be a serious problem in high-speed IM/DD optical transmission systems. On the other hand, in systems with a coherent detection, the PMD effect can be effectively compensated by digital filtering, which will be discussed in Chapter 6. The total pulse broadening due to PMD can be evaluated by using Equation (3.88) for bit rates up to 25 Gb/s (or better to say 25 Gsymbol/s) since the second order PMD term can be neglected. The fraction \( \delta_{s, \text{PMD}} \) of the signal that leaks outside of the bit period due to the PMD impact can be expressed as

\[
\delta_{s, \text{PMD}} = \frac{\sigma_{\text{PMD}}}{T} = \sigma_{\text{PMD}} B
\]  

(4.101)

where \( B=1/T \) is signal bit rate, while \( \sigma_{\text{PMD}} \) is the RMS of the output pulse. The fraction \( \delta_{s, \text{PMD}} \) should be less than a specified value. The power penalty \( \Delta P_{\text{PMD}} \) related to the PMD effect can be evaluated by using formula presented in [26], which is

\[
\Delta P_{\text{PMD}} \approx -10 \log(1 - d_{\text{PMD}})
\]  

(4.102)

where

\[
d_{\text{PMD}} \approx \text{erfc}(\xi) + 2 \sum_{i=1}^{\infty} \text{exp}(-i^2 \xi^2) \left[ \text{erf}(i+1)\xi - \text{erf}((i-1)\xi) \right]
\]  

(4.103)

and

\[
\xi = \frac{T}{2 \tau_0} \frac{\sigma_0}{\sigma}
\]  

(4.104)

We should notice that \( \text{erf}(x) = 1 - \text{erfc}(x) \) is so-called error function, while the complementary error function \( \text{erfc}(x) \) has been defined by Equation (4.53).
Parameters $\sigma$ and $\sigma_0$ are related to the RMS of the output and input pulses, respectively. In we now use Equations (4.97), (4.101), (4.103) and (4.104) for Gaussian shaped pulses, we can obtain that $\xi = 1/(2\sigma)$. Consequently, the parameter $d_{\text{PMD}}$ can be defined through the fraction parameter $\delta_{\text{PMD}}$ as

$$d_{\text{PMD}} = \text{erfc} \left( \frac{1}{2\delta_{\text{PMD}}} \right) + 2 \sum_{i=1}^{\infty} \exp \left( \frac{-i^2}{4\delta_{\text{PMD}}} \right) \left[ \text{erf} \left( \frac{i+1}{2\delta_{\text{PMD}}} \right) - \text{erf} \left( \frac{i-1}{2\delta_{\text{PMD}}} \right) \right]$$  \hspace{2cm} (4.105)

If we assume that input pulse has a Gaussian shape, we can again apply the Equation (4.97) as a criterion of the pulse spreading limit. We can also assume that the first order PMD is a dominant effect in most cases. The fraction parameter $\delta_{\text{PMD}}$ can now be found from Equation (3.88), and it becomes

$$\delta_{\text{PMD}} = \frac{1}{4} \left[ 1 + 2 \frac{\Delta \tau_{\text{P1}}}{\sigma_{\text{w}}} \xi (1-\xi) \right]^{1/2} = \frac{1}{4} \left[ 1 + 64 \frac{\Delta \tau_{\text{P1}}}{\tau^2} \xi (1-\xi) \right]^{1/2}$$  \hspace{2cm} (4.106)

where $\Delta \tau_{\text{P1}}$ defines the differential delay between two principal polarization states over fiber length $L$, while $\xi$ represents the power splitting of the signal between two principal polarization states.

The maximum allowable pulse spreading due to PMD effect can determined by following a recommendation issued by ITU-T in [28], which states that the pulse spreading factor $\delta_{\text{PMD}}$ due to PMD should less than 0.30 for optical power penalty to be below 1 dB. Accordingly, the Equation (3.92) can be applied, so we have that

$$\Delta \tau_{\text{P1}} = \langle D_{\text{P1}} \rangle \sqrt{L} < 0.1 \cdot T$$  \hspace{2cm} (4.107)

where $\langle D_{\text{P1}} \rangle$ presents the mean value of first order PMD parameter expressed in ps/(km)$^{1/2}$. The requirements for the accumulated average first order PMD, and the actual first order PMD group delay (expressed as three times the average value), is summarized in Table 4.4 for several high speed bit rates. The values from Table 4.4 are relevant just for binary IM/DD scheme. However, it they be applied to multilevel modulation schemes where symbol rate, rather bit rate is the relevant parameter. As an example, if QPSK modulation scheme with polarization multiplex is applied the symbol rate will be 25 GSymbol/s, and the actual PMD that can be tolerated is 12 ps (instead of 3 ps valid for binary scheme, as shown in Table 4.4). From design perspective of IM/DD systems, we can say the PMD effect became critical factor for bit rates of 10 Gb/s if the first order PMD is higher than 0.5 ps/(km)$^{1/2}$, while it becomes dominant for binary 40 Gb/s IM/DD scheme if the fibers have the first order PMD that exceeds 0.05 ps/(km)$^{1/2}$. 
Table 4.4
Typical values of the first order PMD tolerance

<table>
<thead>
<tr>
<th>Bit (Symbol) rate in Gb/s</th>
<th>The average PMD tolerated, in ps</th>
<th>Actual PMD tolerated, in ps</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>40</td>
<td>2.5</td>
<td>7.5</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

The impact of the second order PMD can also be evaluated by using Equation (4.106). In such a case, the first order PMD parameter $\Delta \tau_{P1}$ should be replaced with the parameter equal

$$\Delta \tau^2 = \Delta \tau_{P1}^2 + \Delta \tau_{P2}^2$$

(4.108)

where $\Delta \tau_{P2}$ defines the total pulse spreading due to the second order PMD effect in accordance with the Equation (3.96). Finally, in addition to transmission optical fibers, the PMD effect can also occur in other optical elements along the lightwave path. Such elements are optical amplifiers, dispersion compensating modules (DCM), optical switches, etc. It is important to include the impact of the pulse spreading in these elements, especially if bit rate exceeds 10 Gb/s km. The contribution to the pulse spreading due to PMD acquired in different elements can be expressed as

$$\sigma_{PMD, addit} = \left( \sum_i \sigma_{i,PMD}^2 \right)^{1/2}$$

(4.109)

where $\sigma_{i,PMD}$ are PMD contributions from optical elements mentioned above. Typical PMD values of optical modules and functional elements within a single span can be found in the corresponding data sheets, and some of them are listed in Table 4.5. As an example, we can calculate from Equation (4.109) that $\sigma_{PMD, addit}$ will be more than 0.5 ps per optical fiber span, assuming that there is 3-5 optical elements that contribute significantly to the total PMD effect. The generalized formula that includes the entire PMD acquired in transmission fibers and optical modules can be derived from Equations (3.92), (3.96), and (4.109) and written as

$$\sqrt{3\langle D_{P1} \rangle \sqrt{L} + \langle S_{D_{P2}} \rangle \sqrt{L} + \sum_i \sigma_{i,PMD}^2} < 0.3T$$

(4.110)

Table 4.5
Typical values of PMD in optical modules in a single fiber span

<table>
<thead>
<tr>
<th>Module</th>
<th>Actual PMD in ps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical amplifier</td>
<td>0.15 to 0.3</td>
</tr>
<tr>
<td>Dispersion compensating module</td>
<td>0.25 to 0.7</td>
</tr>
<tr>
<td>Optical switch</td>
<td>0.2</td>
</tr>
<tr>
<td>Optical isolator</td>
<td>up to 0.02</td>
</tr>
</tbody>
</table>
4.3.4 The Impact of Nonlinear Effects to System Performance

The transmission quality will be degraded due to impact of nonlinear effects occurring in optical fibers. Some of nonlinear effects, such as selfphase modulation or crossphase modulation, can degrade the system performance through the signal spectrum broadening and the pulse shape distortion. The other effects degrade the system performance either through nonlinear crosstalk (in case of four wave mixing and stimulated Raman scattering), or by signal power depletion (in case of stimulated Brillouin scattering). The power penalty associated to nonlinear effects does not have the same meaning as in cases associated with dispersion or attenuation since the impact of nonlinearities cannot be compensated by an eventual increase of the optical signal power. That is because any increase in signal power will also increase the impact of nonlinearities. Therefore, the power penalty should be considered rather as a measure of the impacts of nonlinearities.

4.3.4.1 Impact of Stimulated Brillouin Scattering (SBS)

In SBS process the acoustic phonons are involved in interaction with light photons, and that occurs over a very narrow spectral linewidth $\Delta\nu_{SBS}$ ranging from 50 to 100 MHz—please recall Equation (2.110). The interaction is rather weak and almost negligible if spectral linewidth is larger than 100 MHz. The SBS process depletes the propagating optical signal by transferring the power to the backward scattered light (i.e. the Stokes wave). The process becomes very intense if the incident power per channel is higher than some threshold value $P_{Bth}$. The threshold value, which is expressed by Equation (2.111), is estimated to be about 7 mW. The SBS penalty can be reduced by either keeping the power per channel below the SBS threshold, or by broadening the linewidth of the light source. The most practical way of the linewidth broadening is through the signal dithering. By applying the spectral linewidth broadening, Equation (2.111) for the SBS threshold becomes [32], [35]

$$P_{Bth} = \frac{2bA_{eff}}{g_{Bmax} L_{eff}} \left(1 + \frac{\Delta\nu_{laser}}{\Delta\nu_{SBS}}\right)$$

(4.111)

where $\Delta\nu_{laser}$ is a broadened value of the source linewidth. As an example, the SBS threshold rises to $P_{Bth} = 16$ mW = 12 dBm if the source linewidth is broadened to be $\Delta\nu_{laser} = 250$ MHz. This value is good enough to prevent any serious impact of the SBS effect.
4.3.4.2 Impact of Stimulated Raman Scattering (SRS)

Since the SRS effect is broadband in nature, its gain coefficient covers much wider wavelength region than the SBS gain. It was explained in Subsection 2.6.2.3 that optical channels spaced up to 125 nm from each other could be effectively coupled by the SRS process, possibly in both directions. This is the main reason why the SRS effect can be used to design an efficient optical amplifier, in spite of the fact that the gain peak \( g_R \approx 7 \cdot 10^{-13} \text{m/W} \) is much smaller than the gain peak associated with the SBS process. The SRS coupling and power transfer occurs from lower wavelength \( \lambda_A \) to a higher wavelength \( \lambda_B \) in a way shown in Figure 4.12, only if both channels are carrying “1” bits at any specific moment, as illustrated in Figure 4.13. The power penalty occurs due to depletion of the signal in the originating channel, and can be expressed as

\[
\Delta P_{\text{SRS}} = -10 \log(1 - \delta_{\text{Raman}})
\]  

(4.112)

where \( \delta_{\text{Raman}} \) represents the coefficient that is proportional to the leaking power portion out of the channel in question. The coefficient \( \delta_{\text{Raman}} \) can be calculated as [32], [33]

\[
\delta_{\text{Raman}} = \sum \frac{g_R i \Delta \lambda_{\text{ch}} P L_{\text{eff}}}{2 A_{\text{eff}}} = \frac{g_R \Delta \lambda_{\text{ch}} P L_{\text{eff}} M(M - 1)}{4 \Delta \lambda_{\text{ch}} A_{\text{eff}}}
\]  

(4.113)

where \( \Delta \lambda_{\text{ch}} \approx 125 \text{ nm} \) is maximum spacing between the channels involved in the SRS process, \( \Delta \lambda_{\text{ch}} \) is the channel spacing, \( P \) is the power per channel, and \( M \) is the number of channels. Equation (4.113) is obtained under the assumption that the powers of all optical channels were equal before the SRS effect took a place. In order to keep penalty below 0.5 dB, it should be \( \delta_{\text{Raman}} < 0.1 \), or

\[
P M(M - 1) \Delta \lambda_{\text{ch}} L_{\text{eff}} = P_{\text{in}} \Delta \lambda_{\text{total}} L_{\text{eff}} < 40 \text{ W} \cdot \text{nm} \cdot \text{km}
\]  

(4.114)

where \( \Delta \lambda_{\text{total}} \) is the total optical bandwidth occupied by all channels. Equations (4.113-4.114) were derived by using results obtained in [30], and [34], and under the assumption that there was no chromatic dispersion involved.
The SRS effect is reduced if there is chromatic dispersion present since different channels travel with different velocities, and the probability of an overlapping between pulses decreases. The coefficient $\delta_{\text{Raman}}$ expressed by Equation (4.113) should be multiplied by factor $\psi \sim 0.5$ if chromatic dispersion exceeds a critical limit (which is 2.5-3.5 ps/nm-km). As an illustration of above conditions, we calculated the power penalty associated with the case when chromatic dispersion is present, which is shown in Figure 4.14. The power penalty was calculated for several representative cases by using Equations (4.112-4.113). The coefficient $\delta_{\text{Raman}}$ was decreased by half to reflect the fact that there is chromatic dispersion present. As we can see from Figure 4.14, the power penalty can be decreased by placing optical channels closer to each other, and by decreasing power per channel.
Since the power decrease will reduce the SNR, it will be beneficial only if the power penalty decreases faster than the power itself. The SRS effect in multichannel system can be neutralized by equalization of the powers associated with individual WDM channels, which is usually done through the dynamic gain equalization applied periodically along the lightwave path.

4.3.4.3 Impact of Four Wave Mixing (FWM)

The FWM effect can be treated as an out-of-band nonlinear crosstalk, and evaluated by using Equation (4.45). However, that negative effect due to FWM cannot be compensated by optical signal power increase. The out of band crosstalk can be measured through the ratio

\[
\delta = \frac{\sum_{\text{other}} P_i}{P_0}, \quad (4.115)
\]

where the nominator in Equation (4.115) contains portions of the intruding powers that originate from all channels except the channel in question, while the denominator refers to the optical power of channel in question. In the FWM process crosstalk components are generated at optical frequencies \( \nu_{\text{out}} = \nu_i + \nu_j - \nu_k \). It occurs in situations when three wavelengths with frequencies \( \nu_i, \nu_j \) and \( \nu_k \) propagate through the fiber, and a phase matching condition among them is satisfied.

The phase matching is defined through the relation between the propagation constants of the optical waves involved in process, which is expressed by Equation (3.192). However, Equation (3.192) commonly takes more practical form given by Equation (3.193), which is referred to so-called degenerate case (such as the WDM transmission.) The optical power of a resultant new optical
wave can be calculated by Equations (3.195) and (3.196), while the power penalty related to the FWM impact can be calculated as

$$\Delta P_{FWM} = -10 \cdot \log(1 - \delta_{FWM})$$

(4.116)

where the crosstalk factor $\delta_{FWM}$ is calculated as

$$\delta_{FWM} = \sum_{i,j,k=1,i \neq j \neq k}^{M} \frac{P_{ijk}}{P_{n}}$$

(4.117)

It should be $\delta_{FWM} < 0.2$ if we would like to keep the power penalty below 1 dB. Equation (4.116) gives us a good idea how to accommodate targeted power penalty by playing with the GVD parameter, channel spacing, and the optical power per channel. As an illustration, we plotted a family of curves related to five different channel spacing: 12.5 GHz, 25 GHz, 50 GHz, 100 GHz, and 200 GHz, which is shown in Figure 4.15. The calculation was aimed to the WDM system with 80 channels. However, just limited number of channels was effectively contributing to the level of the crosstalk since the weight coefficient associated with the intensity of interaction between different WDM channels decreases rapidly with increase of the channel spacing as shown in Figure 4.15. For instance, the impact of the neighboring channels spaced up and down by 50 GHz from the channel in question will be two times stronger than the impact of two channels that are spaced up and down by 300 GHz.

![Figure 4.15 Power penalties due to FWM.](Image)

For 100 GHz and 200 GHz channel spacing, the curves in Figure 4.15 were obtained by calculating the impact of channels that are placed within 600 GHz, either up or down from the channel in question, while for 50 GHz, 25 GHz and 12.5 GHz channel spacing, the curves in Figure 4.15 were obtained by calculating the impact of channels that are placed within 300 GHz. It means that there was the total number of 48 interacting channels for 12.5 GHz channel spacing of 24
interacting channels for 25 GHz channel spacing, 12 interacting channels for 50 GHz channel spacing, 12 interacting channels for 100 GHz channel spacing, and 6 interacting channels with 200 GHz channel spacing. It was assumed that $A_{eff} = 80 \ \mu m$, and $D = 8 \ \text{ps/nm-km}$ ($\beta_2 \approx -10 \ \text{ps}^2/\text{km}$). From the practical perspective, it is important to make sure that chromatic dispersion in transmission fiber lies above a critical value, which is 2.5-3.5 ps/nm-km [32]. This is a precondition that might be followed by the optimization procedure that involves the channel spacing selection and optical power per channel adjustment. The FWM effect will severely degrade the system performance if system operates in vicinity of a zero dispersion region. That fact explains the reason why the nonzero-dispersion shifted optical fibers (NZDSF) have been introduced.

4.3.4.4 The Impact of Selfphase Modulation (SPM)

The Selfphase Modulation (SPM) effect does not cause any crosstalk or power depletion. However, it induces the pulse spectrum spreading, which then interacts with chromatic dispersion and enhances the pulse spreading. The hypothetical power penalty due to SPM can be estimated by using the same approach that was used for chromatic dispersion, i.e.

$$\Delta P_{SPM} \approx 10 \log(\delta_{b,SPM})$$  \hspace{1cm} (4.118)

where $\delta_{b,SPM}$ is the broadening factor due to SPM effect, which can be calculated by using Equation (3.151) as

$$\delta_{b,SPM} = \frac{\sigma_{SPM}}{\sigma_0} = \left[1 + \frac{\sqrt{2L_{eff}}L\beta_2}{2L_{eff}\sigma_0^2} + \left(1 + \frac{4}{6L_{eff}}\frac{L^2\beta_2}{4\sigma_0^2}\right)^{1/2}\right]$$  \hspace{1cm} (4.119)

Figure 4.16 Power penalty due to SPM for 10 Gb/s bit rate.
Equation (4.119) is plotted in Figure 4.16 for 10 Gb/s bit rate, and for several different values of the input power and chromatic dispersion parameter. It was again assumed that the input pulse has a Gaussian shape, and that Equation (4.97) can be applied. Therefore, the parameter $\sigma_\phi$ from Equation (4.119) takes the value $\sigma_\phi = 17.68$ ps. As we can see, even modest value of input optical power can cause considerable power penalties if transmission is done in normal dispersion region (negative chromatic dispersion coefficient). On the other hand, the SPM effect can help to suppress the impact of chromatic dispersion in the anomalous dispersion region. This possibility is further explored through two special techniques known as chirped RZ coding and soliton transmission, which were discussed in Part 3.4.1.3.

4.3.4.5 Impact of Crossphase Modulation (XPM) Effect

Crossphase modulation (XPM) is another nonlinear effect that causes changes in the optical signal phase. This phase shift is the pulse-pattern dependent, and it is converted to the power fluctuations in the presence of chromatic dispersion. Therefore, the signal-to-noise ratio will be diminished through the intensity-like noise, which is also the pulse-pattern dependent. The XPM effect can be reduced by optical power reduction since the root mean square (RMS) of these intensity fluctuations is dependent on the optical power. A rough estimate of XPM effect was done in [29] by assuming that the total phase shift $\phi_{XPM}$ due to XPM should be lower than one radian. Therefore, by calculating the total phase shift from Equations (3.189-3.190), and by assuming that $\phi_{XPM} < 1$, the following restriction for optical channel power is obtained

$$P_{ch,XPM} \leq \frac{\alpha}{\gamma(2M - 1)} \quad (4.120)$$

where $\alpha$ is the fiber attenuation coefficient, $M$ is the total number of channels, while $\gamma$ is the nonlinear coefficient introduced by Equation (3.113). Equation (4.120) returns the value of 0.25 mW if the number of channels is 40, and the value of 0.125 mW if the number of channels is 80. Equation (4.120) will help to establish some reference line, but it does not account for channels walk-off, which occurs because different channels travel with different speeds.

The XPM effect can occur only if two pulses overlap in the time domain, and that means that maximum power per channel will be higher than the value expressed by Equation (4.120). The phase shift due to XPM effect is eventually converted to amplitude variations in the same fashion as in the SPM case. In addition, the bit pattern and power variations between different channels will cause an asymmetric pulse overlapping causing the net frequency shift between different channels, which is then observed as a timing jitter in time domain. Therefore, the system performance degradation due to XPM effect occurs not just
because there are noisy variations in amplitude, but due to timing jitter that has been introduced.

The XPM effect is one of the most serious impairments that can severely degrade performance in multichannel optical transmission systems. More accurate way to evaluate the impact of the XPM effect for different bit patterns is by solving nonlinear wave equations numerically—please refer to Equations (3.199), (3.200) and (3.201). We should mention that novel class of LDPC codes described in Chapter 7 present an efficient tool for minimizing the XPM effect.

4.3.5 The Impact of the Extinction Ratio

All impairments degrade receiver sensitivity by effectively diminishing the positive contribution of the signal power to the SNR and bit-error rate. In some cases, such as the impact of a finite extinction ratio, receiver sensitivity could be recovered by increasing the value of the optical signal by some margin $\Delta P$.

The extinction ratio, introduced by Equation (2.57), is defined by parameter $R_{ex}=P_1/P_0$, where $P_1$ is the power of “1” bit, while $P_0$ is the power of “0” bit. In essence, the extinction ratio is determined by the energy carried by “0” bits, and that is the reason why its inverse value $r_{ex}=1/R_{ex}=P_0/P_1$ is often used in different calculations. The energy carried by “0” bits is relatively high in direct modulation schemes if laser is biased above, or close to the threshold value. On the other hand, the extinction ratio in external modulator schemes is determined by the bias voltage and structure of the modulator.

The $Q$-factor associated with a finite extinction ratio can be calculated by replacing $P_0$ with $r_{ex}P_1$ in Equations (4.57) and (4.58), which leads to

$$Q(r_{ex}) = \left[1 - \frac{r_{ex}}{1 + r_{ex}} \right] \frac{2RP_{ex}}{\sigma_1 + \sigma_0}$$  \hspace{1cm} (4.121)

This value can be now used to find the receiver sensitivity in different scenarios analyzed earlier. As an example, if detection is done by PIN photodiode we can use Equation (4.67) to evaluate the receiver sensitivity through the $Q$-factor. In such a case the receiver sensitivity is given as

$$P_{\text{PIN}}(r_{ex}) = \left[\frac{1 + r_{ex}}{1 - r_{ex}} \right] \frac{\sigma_{\text{d}} Q(0)}{R} = \left[\frac{1 + r_{ex}}{1 - r_{ex}} \right] P_1(0)$$  \hspace{1cm} (4.122)

The power penalty due to a finite extinction ratio now becomes

$$\Delta P_{\text{PIN}}(r_{ex}) = 10 \log \left[\frac{1 + r_{ex}}{1 - r_{ex}} \right]$$  \hspace{1cm} (4.123)
The power penalty evaluation is more complex if APD is used for photodetection since the extinction ratio has an impact on the optimum APD gain. It was first shown in [40] that an optimum APD gain decreases with $r_{ex}$ increase, which leads to the receiver sensitivity degradation. The impact of a finite extinction ratio to receiver sensitivity of APD based optical receivers can be evaluated if we assume that the quantum shot noise is dominant noise factor, and that $\sigma_0 \sim r_{ex} \sigma_1$. With this assumption the $Q$-factor can be expressed as

$$Q(r_{ex} | APD) \approx Q_{indef} \left[ \frac{1 - r_{ex}}{1 + r_{ex}} \right]$$

where $Q_{indef}$ corresponds to an ideal case with an indefinite extinction ratio, which refers to Equation (4.69). The approximate value given by Equation (4.124) can be now inserted in Equation (4.69). The power penalty can be calculated as a difference in receiver sensitivities that correspond to $Q_{indef}$ and $Q(r_{ex})$.

![Figure 4.17 Power penalty due to nonideal extinction ratio.](image)

The power penalty due to nonideal extinction ratio for the PIN and APD based optical receivers is shown in Figure 4.17. The inverse value of the extinction ratio for PIN based optical receivers should be smaller than 0.06 to keep the power penalty below 0.5 dB (dashed line in Figure 4.17), and smaller than 0.12 to keep the power penalty below 1 dB.

### 4.3.6 The Impact of the Intensity Noise and Mode Partition Noise

The intensity noise is caused by intensity fluctuations of the incoming optical signal, with the light source providing main contribution to these fluctuations. The intensity noise can be enhanced by some other effects, such as multiple light reflections along the lightpath, conversion of the phase noise to intensity noise, etc.
Any light intensity fluctuations will be converted to the electrical noise in photodiode and added to the noise components that already exist (i.e. to thermal noise, quantum shot noise, beat noise).

Both the SNR and the receiver sensitivity will be degraded due to impact of the intensity noise. An exact evaluation of the power penalty that is related to the intensity noise is rather complex. However, a simplified approach presented in [17], provides a fairly good estimate of the impact of the overall intensity noise. It was assumed in [17] that the intensity noise power can be simply added to the power of the thermal and shot noise, so that values of noise powers that are related to “1” bits and “0” bits become

\[ \sigma^2_{1} = \langle i^2 \rangle_{1_{\text{int}}} = \langle i^2 \rangle_{1_{\text{int}}} + \langle i^2 \rangle_{0_{\text{int}}} = 2g(M)^2 F(M) I_{\Delta f} + \frac{4\Theta NF_n M}{R_l} + (R P_{\text{in}})^2 \]

\[ \sigma^2_{0} = \langle i^2 \rangle_{0_{\text{total}}} = \langle i^2 \rangle_{0_{\text{int}}} = \frac{4\Theta NF_n M}{R_l} \]

where the noise power due to the intensity fluctuations is expressed as

\[ \langle i^2 \rangle_{\text{int}} = (RP_I)^2 r_{\text{int}}^2 = \frac{(RP_I)^2}{2\pi} \int_{-\infty}^{0} RIN(\omega) d\omega = 2(RP_I)^2 RIN_{\text{laser}} \Delta f \]

where \( RIN(\omega) \) is relative intensity noise (RIN) spectrum defined by Equation (4.15), while \( r_{\text{int}} \) is the parameter introduced by Equation (4.17) that measures the intensity fluctuations. The \( RIN_{\text{laser}} \) parameter in Equation (4.127), which is related to the average value of the RIN spectrum, is smaller than \(-160\) dB/Hz for high quality lasers (i.e. for \( r_{\text{int}} \sim 0.004 \)).

We can calculate the receiver sensitivity degradation due to impact of the intensity noise by assuming that receiver sensitivity is \( P_R = P/2 \), and inserting Equations (4.125-4.127) into Equation (4.57) afterwards. The following expression can be easily obtained

\[ P_R(r_{\text{int}}) = \frac{P_R(0)}{1 - r_{\text{int}}^2 Q^2} \]

(4.128)

It is worth to mention that Equation (4.128) was obtained under assumption that the impact of extinction ratio can be neglected. The power penalty due to impact of the intensity noise can be calculated as

\[ \Delta P_{\text{int}} = 10 \log \left[ \frac{P_R(r_{\text{int}})}{P_R(0)} \right] = 10 \log \left[ 1 - r_{\text{int}}^2 Q^2 \right] \]

(4.129)
The power penalty calculated by Equation (4.129) is shown in Figure 4.18 for three values of the $Q$-factor (i.e., for $Q = 6, 7, 8$). The power penalty is smaller than 0.5 dB for $r_{int}$ values ranging from 0.42 for $Q = 8$, to 0.55 for $Q = 6$. There is very sharp increase in power penalty if the parameter goes above some critical value, which is about $r_{int} = 0.12$ for $Q = 8$, and $r_{int} = 0.15$ for $Q = 6$. The power penalty can become very high and totally degrade the receiver sensitivity if parameter $r_{int}$ lies above these critical values.

There are also other factors, such as reflections, phase noise to intensity noise conversion, mode partition noise, and reflection noise, which also contribute to the total intensity noise. The impact of these noise ingredients can be estimated through an effective intensity noise parameter $r_{eff}$ defined as

$$r_{eff} = \sqrt{r_{int}^2 + r_{ref}^2 + r_{part}^2 + r_{phase}^2}$$

(4.130)

where the individual components on the right side of Equation (4.130) are related to the intensity noise, reflection noise, mode partition noise, and converted phase noise, respectively.

Figure 4.18 Power penalty due to intensity noise.

The evaluation of individual components from Equation (4.130) is not an easy work. The problem is simplified if the DFB lasers are used in combination with single mode fibers, since the intensity noise contribution is basically determined by the first term on the right side. That is because we assume that the
reflection noise is reduced by optical isolators inserted along the lightpath. However, in case when either multimode lasers or VCSEL are used, the contributions from other terms from Equation (4.130) can become dominant. The parameter related to the reflection induced intensity noise can be expressed as

$$r_{ref} \approx \frac{(r_1 r_2)^{1/2}}{\gamma_{iso}}$$

(4.131)

where $r_1$ and $r_2$ are reflection coefficients of two disjoints that were introduced by Equation (4.17), while $\gamma_{iso}$ is the attenuation coefficient of the optical isolator. This reflection coefficient becomes $r_{ref} \approx 3.6\%$ or $-14.4$ dB if it is related to connectors that include the air/glass interface, and if there is no optical isolator employed. However, it becomes comparable or less than $-55$ dB if there is an optical isolator in place since the attenuation of an optical isolator is usually higher than 40 dB.

The impact of the intensity noise introduced by the mode partition effect can be expressed through parameter $r_{part}$ as [17]

$$r_{part} \approx \left(\frac{k}{\sqrt{2}}\right)\left[1 - \exp\left(-\left(\pi B LD \sigma_{\lambda}\right)^2\right)\right]$$

(4.132)

where $k$ is a coefficient that vary in the range 0.6-0.8, $B$ is signal bit rate, $L$ is the transmission distance, $D$ is chromatic dispersion coefficient, while $\sigma_{\lambda}$ represents the spectral linewidth of the light source. Please recall from Section 4.1.2 that the impact of the mode partition noise can be almost entirely suppressed if the product $BLD\sigma_{\lambda}$ is less than 0.075, which corresponds to the value $r_{part} \approx 0.15$.

4.3.7 The Impact of the Timing Jitter

The timing jitter arises due to fluctuations of the time clock recovery instants since the sampling time is not quite stable [1]. In fact, the clock fluctuates in time around specified instants due to noisy nature of the incoming signal. This will cause the sampling intervals to fluctuate around the bit center, which means that the sampled value of the signal will not always be aligned with the signal maximum. The timing jitter is measured by a random time variable $\Delta t$, which reflects the fluctuation of any specific sampling instant from the bit center. The impact of the timing jitter at the decision point is similar to the impact of the intensity noise since any variations in sampling times are eventually converted to intensity variations of samples. Such a negative impact can be suppressed by increase in the signal power that is equal to the induced power penalty.

The impact of the jitter to receiver sensitivity can be analyzed by using rather complex numerical methods, such as one presented in [26]. On the other hand, there is an approximate approach proposed in [17] that offers good understanding
of the statistics involved in the timing jitter process. It was assumed in [17] that
noise parameters from Equation (4.57) can be expressed as

\[
\sigma_i^2 = \langle i_i^2 \rangle_{\text{total}} = \langle i_i^2 \rangle_{\text{th}} + \langle i_i^2 \rangle_{jitt} = \frac{4k\Theta NF_{\text{w}}\Delta f}{R_t} + \langle \Delta i_i^2 \rangle
\]

(4.133)

\[
\sigma_N^2 = \langle i_N^2 \rangle_{\text{total}} = \langle i_N^2 \rangle_{\text{th}} = \frac{4k\Theta N F_{\text{w}}\Delta f}{R_t}
\]

(4.134)

where \( \langle i_i^2 \rangle_{jitt} \) is the power of the noise component due to timing jitter, while
\( \langle \Delta i_{jitt} \rangle \) is standard deviation of current fluctuations induced by time variations \( \Delta t \).

It is clear that the pulse shape will have a big impact on the timing jitter that is
generated. The current fluctuations can be evaluated more precisely if we know
the function \( h_{\text{out}}(t) \) that governs the shape of the current signal.

In general case we can assume that the function \( h_{\text{out}}(t) \) takes the raised cosine
shape—please refer to Section 10.9. In order to evaluate the noise power, it is
necessary to find the variance \( \langle \Delta i_{jitt}^2 \rangle \) of the stochastic variable \( \Delta i_{jitt} \). It was
assumed in [17] that \( \Delta i_{jitt} \) follows the Gaussian distribution, and that its variance
can be calculated as

\[
\langle \Delta i_{jitt}^2 \rangle = 8I_i^2 \left[ (B\sigma_{\Delta})^2 (\pi^2 / 3 - 2) \right]^{1/2}
\]

(4.135)

where \( \sigma_{\Delta} \) is the standard deviation of time fluctuations \( \Delta t \). The receiver sensitivity
can be evaluated by inserting Equations (4.133-4.135) in expressions given by
Equations (4.57), (4.58), (4.67), which eventually leads to

\[
Q = \frac{I_c - \langle \Delta i_{jitt} \rangle}{\sigma_i + \sigma_o} = \frac{I_c [1 - (B\sigma_{\Delta})^2 (2\pi^2 / 3 - 4)]}{8I_i^2 \left[ (B\sigma_{\Delta})^2 (\pi^2 / 3 - 2) \right]^{1/2} \left[ \frac{4k\Theta NF_{\text{w}}\Delta f}{R_t} \right]^{1/2}}
\]

(4.136)

and

\[
P_k(\sigma_{\Delta}) = P_k(0) \frac{1 - (B\sigma_{\Delta})^2 (2\pi^2 / 3 - 4)}{[1 - (B\sigma_{\Delta})^2 (2\pi^2 / 3 - 4)]^2 - 8Q^2 \left[ (B\sigma_{\Delta})^2 (\pi^2 / 3 - 2) \right]^2}
\]

(4.137)

The power penalty due to impact of timing jitter can be calculated from Equation
(4.137) as

\[
\Delta P_{\text{nw}} = 10 \log \left[ \frac{P_k(\sigma_{\Delta})}{P_k(0)} \right]
\]

(4.138)

The power penalty calculated by Equations (4.137) and (4.138) is below 0.5 dB if
the RMS value of the jitter is lower than 10% of the bit interval (i.e. if \( \sigma_{\Delta}B < 0.1 \)).
However, if the RMS value of the jitter were higher than 20% of the bit interval it would produce indefinite power penalties. Equations (4.137-4.138) are good for approximate evaluation of the power penalty. More precise calculations, such as one presented in [26], shows that the power penalty is even higher than the one estimated by Equation (4.138). However, the criteria $\sigma_{\Delta B} < 0.1$ is quite valid to be used as guidance in transmission system design.

### 4.3.8 The Impact of Signal Crosstalk

The crosstalk noise is related to multichannel systems, and can be either *out-of-band* or *in-band* in nature, as discussed in Section 4.1.9. Out-of-band crosstalk occurs when the fraction of the power of an optical channel spreads outside of the channel bandwidth and mixes with the signals of neighboring channels. Therefore, optical receiver of any specific optical channel can capture the interfering optical power and convert it to the electrical current, which can be expressed as

$$I_{\text{cross, out}} = \sum_{m} R P_{n} X_{n} = R P \sum_{m} X_{s}$$  \hspace{1cm} (4.139)

where $R$ is photodiode responsivity, $P$ is optical power per channel, while $X_{n}$ is portion of the $n$-th channel power that has been captured by the optical receiver of the $m$-th optical channel. For convenience purpose, the parameter $X_{n}$ can be identified as “the crosstalk ratio”. The crosstalk current, or better to say the crosstalk noise, can be treated as an intensity noise. Consequently, the impact of the out-of-band crosstalk noise can be evaluated by applying Equation (4.129) to this specific case. Accordingly, the following set of equations can be established

$$I_{\text{cross, out}}^2 = \left[ \sum_{m} X_{s} \right]^2$$  \hspace{1cm} (4.140)

$$\left< I_{\text{cross, out}}^2 \right> = (R P_{n})^2 I_{\text{cross, out}}^2 = (R P_{n})^2 \left[ \sum_{m} X_{n} \right]^2$$  \hspace{1cm} (4.141)

$$\Delta P_{\text{cross, out}} = -10 \log \left[ 1 - \left< I_{\text{cross, out}}^2 \right> O^2 \right]$$  \hspace{1cm} (4.142)

The same approach can be used when considering the impact of in-band crosstalk noise to the receiver sensitivity. We should recall that the in-band crosstalk noise occurs when interfering optical power has the same wavelength as the wavelength of the optical channel in question. The following set of equations can be established by applying Equation (4.129) to this specific case

$$I_{\text{cross, in}} = 2 R P \sum_{m} X_{s}$$  \hspace{1cm} (4.143)
Since both out-of-band and in-band crosstalk can be identified as the intensity noise-like impairments, the total impact of the crosstalk can be evaluated by using an equivalent noise parameter

\[ r_{cross}^2 = \sqrt{r_{cross, out}^2 + r_{cross, in}^2} \]  

(4.146)

This parameter can be inserted in Equation (4.145) in order to evaluate the receiver sensitivity degradation due to the total crosstalk effect, so that it becomes

\[ \Delta P_{cross} = -10 \log \left[ 1 - r_{cross}^2 Q^2 \right] \]  

(4.147)

We can evaluate the total impact of the crosstalk noise by assuming that there is a single crosstalk contribution that dominates in the sums in Equations (4.145) and (4.147), while the impact of other terms can be neglected. This assumption basically means that this single crosstalk acts as an equivalent crosstalk ratio \( X_{eq} \).

The concept of an equivalent crosstalk ratio was used in Equations (4.142), (4.145) and (4.147) to calculate the crosstalk related power penalties. It was shown in [33] that the impact of the in-band crosstalk is the main contributor to the equivalent crosstalk parameter. Also, the power penalty due to out-of-band crosstalk causes more than 1 dB power penalty if crosstalk ratio \( X_{eq} \) becomes larger than 6.7\%, which means that the power penalty will be lower than 1 dB if the interfering optical power is at least 11.7 dB below the power level associated with channel in question. At the same time, the power penalty due to in-band crosstalk will be higher than 1 dB power if crosstalk ratio \( X_{eq} \) becomes larger than 0.85\% (the interfering optical power is 20.65 dB below the signal level.) If power penalty is kept below 0.5 dB, out-of-band crosstalk should be at least 13.4 dB below the signal in question, while in-band crosstalk should be at least for 23.5 dB lower than the signal.

### 4.3.9 Impact of Raman Amplification to Signal Distortion

In parallel with the signal amplification, Raman amplifier also induces signal impairments. These impairments are related to the amplified spontaneous emission (ASE) noise, double Rayleigh backscattering (DRB) noise, and signal distortion due to nonlinear Kerr effect [41]. Two dominant noise sources (the ASE noise, and the DBR noise) are illustrated in Figure 4.19.
The power of ASE noise can be found by applying a general formula given by Equation (4.3), i.e.

\[
P_{\text{ASE}} = 2n_{sp} h v_S \Delta v_R = \frac{2h v_s \Delta v_R}{1 - \exp \left( \frac{h v_S}{kT} \right)}
\]

where \( v_S = \omega_S / 2\pi \) is the optical frequency of the signal, while \( n_{sp} \) is the spontaneous emission factor that is determined by the thermal energy. The factor 2 in Equation (4.148) accounts for two polarization modes within the spontaneous emission. The power of spontaneous emission is much smaller than that in EDFA, due to distributed nature of the Raman amplification. The difference between EDFA and a Raman amplifier is that \( n_{sp} \approx 1.13 \) in Raman amplifier, as there is always a full population inversion during the simulated amplification. One can see from Equation (4.148) that the ASE noise is a broadband in nature and the white-noise like.

Generation of double Rayleigh backscattering noise is also illustrated in Figure 4.19. This effect occurs due to Rayleigh scattering, which is normally present in all optical fibers, but in much smaller extent. We should recall that the major part of the scattered light goes backward, while a smaller part takes a forward direction and adds to the signal. If there is a distributed amplification process, the photons due to the Rayleigh scattering are multiplied. In this case we can consider that the forward propagating portion of the Rayleigh scattering noise is added to the ASE noise, thus contributing to the increase of the total noise level. Generally speaking, even amplified, this portion is not a serious issue. However, what becomes an issue relates to multiple reflections of the enhanced Rayleigh scattering. In this case, the optical fiber is acting as a weak distributed mirror. The backward scattering eventually turns to the forward one, thus creating crosstalk to the propagating optical signal. This crosstalk noise, which is referred as the double Rayleigh backscattering (DRB) noise, accumulates in long-haul transmission systems that have a large number of amplifiers. Such systems are, for example, the
submarine transmission systems, in which the DRB noise can reach a level that severely degrades the system performance.

The main difference between ASE and DRB noise components is in the following: (i) ASE is an additive noise independent from the signal; (ii) DRB is a replica of the signal optical spectrum and is also known as multipath interference (MPI). The increase in DBR noise is the main reason why gain in Raman amplifiers can not be increased above some critical value. Namely, in Raman amplifier, the short lifetime of the electrons at the upper state results in instantaneous gain which leads to coupling of the pump and the signal powers, which is favorable from signal gain perspective. However, pump fluctuations are also coupled with the signal, which increases the overall noise. This is suppressed by backward pumping configurations which is less efficient in terms of the Raman amplifier gain, but more favorable from the noise perspective.

The power of the DRB noise can be calculated as [41]

\[
P_{DRB} = P_s(0) G_R e^{G(z)dz} G(z) dz
\]  

(4.149)

where \( P_s(0) \) is launched signal power, \( G_R \) in Raman gain often known as on-off gain, \( r \) is the Rayleigh backscattering coefficient (\( r \approx 1.03 \cdot 10^{-4} \text{ km}^{-1} \)), \( G(0,z) \) is net gain over distance ranging from 0 to \( z \) (the net gain equals the amplification minus attenuation), \( L_{span} \) is the fiber span length that precedes to distributed Raman amplifier. In case of lumped Raman amplifiers, just the ASE noise will be present, while the DRB noise can be neglected. The DRB noise is not white-like, but it is rather Gaussian-like and signal-related.

The Raman amplification process also introduces the signal impairment that comes from the impact of Kerr effect. That impact can be evaluated by calculating the integrated nonlinear phase along the length \( L_{span} \), which is

\[
\Phi_{DRA} = \int_{0}^{L_{span}} \gamma P(z) dz
\]  

(4.150)

where \( \gamma \) is the nonlinear Kerr coefficient introduced by Equation (3.113), while \( P(z) \) refers to the evolution of the signal power along \( z \)-axis. The power evolution in distributed Raman amplifiers (DRA) is more complex than a simple attenuation that occurs in transmission optical fibers. In general case, DRA is bi-directionally pumped, while the ratio between the forward and backward pump powers is a factor that determines the character of the optical signal evolution along the span length. If there were no any pumping, the integrated phase given by Equation (4.150) would take a linear form, which can be calculated as

\[
\Phi_{pass} = P_s(0) \int_{0}^{L_{span}} \gamma \exp(-\alpha_S z) dz = \gamma P_s(0) \frac{1 - \exp(-\alpha_S L_{span})}{\alpha_S}
\]  

(4.151)
The index “pass” in Equation (4.151) stands for “passive fiber”, which means that phase is calculated when there is no pumping at all. The impact of nonlinear index can be measured through the ratio

\[
R_{\text{nl}} = \frac{\Phi_{\text{DRA}}}{\Phi_{\text{pass}}} = \frac{\alpha_s \int_0^L G(0,z)dz}{1 - \exp(-\alpha_s L)}
\]  

(4.152)

Both, the ASE noise and the DRB noise, will accompany the signal at the photodiode area. They will beat with the signal to produce the beat-noise electrical components during photodetection process. The dominant components of the beat-noise will come from the signal-DRB beating and the signal-ASE beating. It was found in [41] and [43] that the following approximations for noise variances can be used to evaluate the beat-noise components

\[
\sigma_{S,\text{ASE}}^2 = 2R^2P_{\text{ASE}}P_{S,1} \Delta f
\]  

(4.153)

\[
\sigma_{S,\text{DBR}}^2 = 2R^2P_{\text{DBR}}P_{S,1} \Delta f
\]  

(4.154)

where \( R \) is photodiode responsibility, \( P_{S,1} \) is the signal power related to “1” bit while \( P_{\text{ASE}} \) and \( P_{\text{DBR}} \) represent powers of the ASE and DRB noise components, given by Equations (4.148) and (4.149), respectively. The equivalent noise figure of distributed Raman amplifier (such as the one shown in Figures 2.9, and 4.20b) can be calculated as [41]

\[
NF_{\text{DRA}} = \frac{\text{SNR}_{\text{opt}}}{\text{SNR}_{\text{opt}}} = \frac{1}{G_R\Gamma} \left( \frac{P_{\text{ASE}}}{B_{\text{op}}h v_s} + \frac{5P_{\text{DBR}}}{9h v_s \sqrt{(\Delta f^2 + B_{\text{op}}^2 / 2)}} + 1 \right)
\]  

(4.155)

where, \( v_s \) is the optical frequency, \( \Delta f \) is the frequency bandwidth of the electrical receiver, \( B_{\text{op}} \) is the bandwidth of the optical filter, \( G_R \) is the Raman on-off gain, while \( \Gamma \) is the loss at the optical fiber span \( \Gamma = \exp(-\alpha_S L_{\text{pan}}) \).

Raman on-off gain \( G_R \) is the key parameter in evaluation of the equivalent noise figure in Equation (4.155). It can be calculated as [41]

\[
G_R(z_1,z_2) = \frac{C_{r}(\lambda_s,\lambda_p)}{\alpha_p} \left[ P \left( \frac{1}{\alpha_p} \left( e^{-\alpha_p(z_2-z_1)} - e^{-\alpha_p(z_1-z_2)} \right) \right) P \left( e^{-\alpha_p z_2} - e^{-\alpha_p z_1} \right) \right]
\]  

(4.156)

where \( \alpha_p \) is fiber attenuation for the pump (\( \alpha_p \approx 0.25 \text{ dB/km} \)), \( z_1 \) and \( z_2 \) are the points along the transmission fiber, while \( P_s \) and \( P_f \) are pump powers in backward and forward directions, respectively. Parameter \( C_{r}(\lambda_s,\lambda_p) \) is known as Raman gain efficiency (gain per unit length and unit power), which is around 0.42 (W·km)^{-1} for standard single mode optical fiber. It was shown in [41] that the impact of \( P_{\text{DBR}} \) can be neglected as compared with \( P_{\text{ASE}} \) if \( G_R \) is kept below 30 dB and proper distribution between forward and backward pumping is applied.
The equivalent noise figure of Raman amplifier can also be evaluated by measuring on-off gain $G_R$ and $P_{ASE}$ and $P_{DRB}$ powers. It is important to mention that the value of $NF_{DRA}$ can be negative as a result of distribution character of Raman gain. What the negative value really means is that the net-result of the Raman amplifier deployment is the SNR enhancement since the employment of Raman amplifiers will prevent the signal to coming closer to the noise level.

The benefit of distributed Raman amplification can be estimated in a real-case scenario when Raman amplifier is deployed in combination with EDFA, as shown in Figure 4.20. Since Raman amplifier includes the entire fiber span, it is useful to introduce noise figure of the fiber span section to make comparison between different cases more clear. The overall noise figure of the single fiber span in case when Raman amplifier is “off” (only EDFA amplifiers are active), and “on” can be calculated by using Equation (4.36) for concatenated optical amplifiers, so we now have that

$$NF_{\text{link}} = \begin{cases} 
NF_{\text{passive}} = \frac{NF_{\text{EDFA}}}{G} + 1 - \Gamma_{\text{loss}} \\
NF_{\text{active}} = NF_{\text{DRA}} + \frac{NF_{\text{EDFA}} - 1}{G \Gamma} + 1 - \Gamma_{\text{loss}}
\end{cases} \tag{4.157}$$

where $NF_{\text{EDFA}}$ is the noise figure of the EDFA amplifier, $\Gamma$ is optical fiber span loss, while $\Gamma_{\text{loss}}$ is the loss caused by some other elements, such as dispersion compensation module (DCM) inserted at the amplifier site.

In general case each span might operate at different launched powers due to different Kerr nonlinearities, but we can assume that the Kerr effect is the same for all spans. The improvement in the SNR after $N$ spans can be calculated as

$$R_{\text{DRA}} = \frac{SNR_{\text{pass}}}{SNR_{\text{DRA}}} = R_{\text{NL}} \frac{N(\frac{NF_{\text{DRA}} - 1}{N} + 1)}{N(\frac{NF_{\text{pass}} - 1}{N} + 1)} \approx R_{\text{NL}} \frac{NF_{\text{DRA}}}{NF_{\text{pass}}} \tag{4.158}$$

It was shown in [41] that previous ratio can be optimized by optimizing the ratio of backward pumping to forward pumping, and it was found that the mix of 30-35% of forward pumping and 65-75% of backward pumping can bring the largest benefit. The improvement of the signal-to-noise ratio is illustrated in Figure 4.21 for two different values of the Raman gain ($G_{R1} < G_{R2} < 30 \, \text{dB}$), where contribution from DRB has been neglected. We can see from Figure 4.21 that signal does not come close to the noise level, as it would be if just EDFA were deployed. In this case, the Raman amplifier acts as a low-noise preamplifier to EDFA, and helps to reduce both the gain and the noise figure of EDFA. On the other hand, it is also beneficial for Raman amplifiers to operate in combination with EDFA since it allows them to stay below the double Rayleigh backscattering (DRS) limit.
Figure 4.20 Deployment of Raman amplifiers: (a) passive fiber with no Raman amplification, (b) active fiber Raman amplification

Figure 4.21 Signal to noise ratio improvement due to Raman gain

The total yield of Raman amplification is proportional to the improvement factor $R_{DRA}$ from Equation (4.158). This yield can be also transferred to the optical
Noise Sources and Channel Impairments

Domain and considered to be the optical power gain or a “negative optical power penalty”. For this purpose we can apply Equation (4.92), and that leads to the following relation

\[ \Delta P_{\text{DRA}} = R_{\text{DRA}} \frac{2A\Gamma}{B_{\text{op}}} \]  

(4.159)

where \( \Delta P_{\text{DRA}} \) is the optical power gain that is brought to the system by employment of distributed Raman amplifiers, \( A\Gamma \) is electrical filter bandwidth, while \( B_{\text{op}} \) is the bandwidth of an optical filter. The value of \( \Delta P_{\text{DRA}} \) can offset some power penalties and to relieve power margin requirements that were needed before the Raman amplifier was employed. In such a way, it plays the role of a “negative power margin”.

As already mentioned, an efficient Raman amplification can also be applied over shorter lengths of the optical fiber if cross-sectional area is smaller. In such a case, we are talking about lumped Raman amplifiers. This scheme can been used to compensate for the losses in a dispersion compensating fiber (DCF) by introducing backward pumping through it, to work in combination with distributed Raman amplifiers (so-called all-Raman combination) to enhance the power level, or to amplify the signals covered by the S-band as in [44]. It was also shown in [44] that dispersion-compensation fibers (DCF) can be used as an effective fiber medium for lumped Raman amplification since they have smaller effective area and higher gain. The way the lumped Raman amplifiers are analyzed is similar to one applied to EDFA.

### 4.3.10 Impact of the Accumulated Noise

We can assume that there is a number of cascaded optical amplifiers along the lightpath spaced \( L_{\text{span}} \) km apart, where parameter \( L_{\text{span}} \) defines the span length, as shown in Figure 4.22. The span loss between two amplifiers is \( \Gamma_{\text{span}} = \exp(-\alpha L_{\text{span}}) \), assuming that the fiber attenuation coefficient is \( \alpha \). The purpose of each in-line optical amplifier is to provide the gain to incoming optical signal just enough to compensate for the loss at the previous span. However, each amplifier also generates some spontaneous emission noise in parallel with the optical signal amplification. Both the signal and the spontaneous emission noise continue to propagate together to be amplified by the following optical amplifiers. The buildup of amplifier noise is the most critical factor in achieving specified systems performance when dealing with longer distances (lightpaths).

The amplified spontaneous emission (ASE) noise will be accumulated and increased after each subsequent span, thus contributing to the total noise and to deteriorating the SNR. In addition, the ASE power will contribute to the saturation of optical amplifiers, thus reducing the amplifier gain and the amplified signal level. This process is also illustrated in Figure 4.22. As we see, the optical signal that starts from some launched level will experience attenuation over the span...
length before it is enhanced by an optical amplifier. Optical amplifier helps to restore the output power level, but generates the spontaneous emission noise by itself, and amplifies the spontaneous emission noise coming from previous amplifier. The ASE noise level will increase after each amplifier, and will eventually force optical amplifiers into saturation regime. Since the output power stays on the same level, the increase in ASE will diminish the optical signal level. The end result will be a decrease in the optical signal to noise ratio (OSNR). Described picture can be transferred to a mathematical relation that connects the signal and noise parameters with respect to the transmission length.

![Figure 4.22 Transmission system employing optical amplifiers](image)

We can assume that the amplifier gain $G$ is adjusted to just compensate for the span loss $\Gamma_{span}$. Otherwise, if the gain was larger than the span loss, the signal power would increase gradually throughout the amplifier chain forcing the amplifier saturation regime. Please recall that the saturation means that the amplifier gain drops as input power increases. At the end of the settlement process, the amplifiers will enter into the saturation regime, while the total gain will drop from its initial value $G_0 = G_{sat}$ to a saturated value $G_{sat}$ — please refer to Equation (2.94). The output power from optical amplifier is determined by the saturated value $P_{o,sat}$ which is given by Equation (2.93). From practical point of view, it is important to consider just a spatial steady-state condition that was reached in long-haul transmission systems, in which both the saturated output power $P_{o,sat}$ and the gain $G_{sat}$ remain the same and govern the signal amplification process. The output power per channel in this case will be $P_{out} = P_{o,sat}/M$, where $M$ is the number of optical channels that are multiplexed and amplified together. As
already mentioned, the OSNR gradually decreases along the lightpath, since the accumulated ASE noise gradually makes up a more significant portion of the limited total power from an optical amplifier.

The steady-state gain, or saturated gain, will be slightly smaller than the span signal loss, due to added noise at each amplifier point. Therefore, the best engineering approach is to choose a saturated gain that is very close to the span loss. In such a case, the following balance between the signal and accumulated noise can be established

\[ P_{o, \text{sat}} \exp(-\alpha L_{\text{span}}) G_{\text{sat}} + P_{\text{op}} = P_{o, \text{sat}} \]  

(4.160)

where \( P_{\text{op}} \) is the ASE power defined by Equation (4.37). In this case, the ASE power can be expressed as

\[ P_{\text{op}}(v) = 2S_{\text{sp}}(v) B_{\text{op}} = (G_{\text{sat}} - 1) NF_{\text{op}} h v B_{\text{op}} \]  

(4.161)

where \( S_{\text{sp}}(v) \) is spectral density of the spontaneous emission that is given by Equation (4.34), \( v \) is the optical frequency, \( B_{\text{op}} \) is the optical filter bandwidth, \( NF_{\text{op}} = 2n_{\text{sp}} \) is the noise figure of the optical amplifier, while \( n_{\text{sp}} \) is the spontaneous emission factor defined by Equation (2.15). It is important to recall that the saturated gain also satisfies Equation (2.94), which can be rewritten as

\[ G_{\text{sat}} = 1 + \frac{P_{\text{sat}}}{P_{\text{in}}} \ln \frac{G_{\text{sat}}}{G_{\text{max}}} = 1 + \frac{P_{\text{sat}}}{P_{o, \text{sat}}} \exp(-\alpha L_{\text{span}}) \ln \frac{G_{\text{max}}}{G_{\text{sat}}} \]  

(4.162)

Equation (4.161) can be now used to evaluate the total noise power that is associated with a steady state reached at the end of the transmission line, and it becomes

\[ N \cdot P_{\text{op}}(v) = N(G_{\text{sat}} - 1) NF_{\text{op}} h v B_{\text{op}} = (L / L_{\text{span}})(e^{-\alpha L_{\text{span}}} - 1) NF_{\text{op}} h v B_{\text{op}} \]  

(4.163)

where \( N = L / L_{\text{span}} \) represent the number of spans at the transmission line, which is equal to the number of optical amplifiers employed along the line. Please notice that the saturated gain value was chosen just to compensate for the signal loss at the preceding span.

4.3.10.1 The OSNR at the end of Transmission Link

The optical signal to noise ratio, calculated per channels basis at the end of the transmission line, can be expressed as
where \( P_{\text{tot}} \) is the total launched optical power at the optical amplifier output, while \( P_{\text{ch}} \) is the launched power within an individual optical channel. In order to satisfy specified OSNR, the launched channel power should satisfy the equation

\[
OSNR = \frac{P_{\text{sat}}}{M - NF_{\text{no}} h v B_{sp} (e^{\alpha L_{\text{span}}} - 1) L / L_{\text{span}}} = \frac{P_{\text{ch}} - NF_{\text{no}} h v B_{sp} (e^{\alpha L_{\text{span}}} - 1) N}{NF_{\text{no}} h v B_{sp} (e^{\alpha L_{\text{span}}} - 1) N} \quad (4.164)
\]

If we did not worry about nonlinearities, we would maximize the power per channel to easily achieve to requirement given by Equation (4.165). However, the real story is different since any power increase will boost the nonlinear effects. Equation (4.165) can be converted to decibel units if the operator \( 10 \log(\cdot) \) is applied to both sides, in which case it becomes

\[
P_{\text{ch}} \geq (OSNR + 1) \left[ NF_{\text{no}} h v B_{sp} (e^{\alpha L_{\text{span}}} - 1) N \right] \approx OSNR \left[ NF_{\text{no}} h v B_{sp} (e^{\alpha L_{\text{span}}} - 1) N \right] \quad (4.165)
\]

The OSNR value can be now used as an input parameter to calculate the total length \( L = N \cdot L_{\text{span}} \) than can be achieved. That value can be obtained by using relation between OSNR and \( Q \)-factor, given by Equations (4.91-4.92). As an example, we used Equation (4.92) to express the connection between OSNR and \( Q \)-factor in Equation (4.167) above. Therefore, we can first calculate OSNR for a specified \( Q \)-factor, and then use Equation (4.167) to evaluate the other transmission parameters. These parameters include: launched optical power per channel, noise figure of optical amplifiers, number of fiber spans, optical fiber bandwidth, and optical loss per fiber span. It is important to mention that although the Equation (4.1670 is obtained having IM/DD scheme in mind, it can be applied to any modulation/detection scenario. What we need in such case is to adjust OSNR (or \( Q \)) requirements, as well as \( B_{sp} \) and \( \Delta f \) parameters in Equation (4.167) to situation that corresponds a specific modulation/detection scenario.

As we can see from Equation (4.167), the OSNR for a specified link length can be increased by increasing the output optical power or by decreasing the number of optical amplifiers. In addition, The OSNR can be improved by decreasing the amplifier noise figure and optical loss per fiber span. However, the number of amplifiers can be decreased only if the span length is increased, which means that the total signal attenuation will be also increased. Therefore, the overall picture becomes more complex, and some tradeoffs may be needed.
4.3.10.2 OSNR for Soliton Transmission

The analysis performed above did not consider impact of nonlinear phase and possible soliton formation. As for solitons, which were introduced in Section 3.4.1.3, any amplification of soliton pulses will be followed by generation of amplified spontaneous emission (ASE) that will be mixed with dispersive waves during the soliton regeneration. The overall transmission performance degradation is caused by the generated noise and by fluctuations in the pulse energy, since both of these effects will decrease the SNR. In addition, the performance degradation will come through the timing jitter, which is induced by frequency fluctuations. The total energy fluctuations during the soliton amplification process were estimated in [29]. It was found that the OSNR could be expressed as

$$\text{OSNR}_{\text{soliton}} = \frac{E_0}{2NS_{sp}h\nu(G-1)}$$

where $E_0$ is the energy of the pulse calculated as an integral of the optical power over time, $N$ is the number of optical amplifiers along the line, $S_{sp}$ is the spectral density of the ASE noise, $n_{sp}$ is spontaneous emission factor, $h$ is the Planck’s constant, while $\nu$ is the optical frequency.

Equation (4.168) applies to standard soliton systems with in-line amplification, but with no any real dispersion management in place. The dispersion management will improve the $\text{OSNR}_{\text{soliton}}$ since the pulse energy is higher than the energy of standard solitons. The $\text{OSNR}_{\text{soliton}}$ improvement is proportional to $F_{dm}^{1/2}$, where $F_{dm}$ is the energy enhancement factor given by Equation (3.172). The timing jitter due to change in soliton frequency, which affects the speed at which soliton propagates through the optical fiber, is the most serious problem that limits the overall capabilities of soliton based transmission systems. This jitter is referred as the Gordon-Haus jitter [45]. The standard deviation $\sigma_t^2$ of the timing jitter introduced in standard soliton systems is [17]

$$\frac{\sigma_t^2}{\tau_0^2} = \frac{S_{sp}L}{9E_{\nu,\text{in}}F_{dm}L_{\text{span}}}$$

where $\tau_0$ is the initial soliton pulse width, $L$ is the total transmission length that is equal $L = N\cdot L_{\text{span}}$, where $L_{\text{span}}$ is the span length (or amplifier spacing), $E_{\nu,\text{in}}$ is the soliton energy given by Equation (3.170), while $L_D$ is the dispersion length equal $L_D = \tau_0^2/|\beta_2|$. On the other hand, the standard deviation of the timing jitter introduced in dispersion managed soliton systems can be expressed as

$$\frac{\sigma_t^2}{\tau_{\text{m}}^2} \approx \frac{S_{sp}L^3}{3E_{\nu,D_{\text{in}}}F_{dm}L_{\text{span}}}$$
where $E_0$ is the soliton energy at the output of transmitter, $T_m$ is the minimum pulse width occurring during transmission, while $L_{D,\text{dm}}=T_m^2/|\beta_2|$. Please recall that the soliton pulse width oscillates in dispersion managed case, and that the minimum occurs at the middle of the anomalous GVD section. By comparing equations (4.169) and (4.170), and by assuming that $T_m \sim \tau_0$, one can see that the timing jitter is reduced in dispersion managed solitons by factor $(F_{\text{dm}}/3)^{1/2}$.

It is important to mention that so-called soliton “self-frequency shift” is also observed in soliton based transmission systems. This effect occurs due to inter-pulse Raman scattering effect, which is enabled by the broad spectrum of the soliton pulse. The high frequency components of the pulse shift energy to lower frequency components, and cause a continuous downshift in the soliton carrier frequency. The soliton self-frequency shift is more intense for higher bit rates since narrower soliton pulses produce much broader pulse spectrum. This can be better understood if we recall that solitons are the transform limited pulses that satisfy Equation (3.128).

Timing jitter that occurs in soliton transmission should be carefully controlled to minimize its negative impact. It can be done by placing optical filters after each amplifier since it will limit the ASE noise power, while simultaneously increasing the value of $OSNR_{\text{soliton}}$. The growth of the timing jitter can be reduced by so-called sliding optical filters, which shifts the central frequency along the link [46]. Such a frequency sliding follows the self-frequency shift and ensures that the maximum portion of the noise power has been removed.

Solitons regime can also be supported in WDM transmission systems. However, it is necessary to account the effect of pulse collisions among solitons belonging to different WDM channels. During such a collision, the crossphase modulation (XPM) effect induces the time dependent phase shift, which leads to change in the soliton frequency. Such a frequency change will either speed up, or slow down the pulses. At the end of collision process, each pulse eventually recovers the in frequency and speed, but the timing jitter that has been induced will remain in place. The timing jitter would not be a factor if the data stream were consisted just of “1” bits since the change will affect all pulses equally. However, since there is a random content of “1’ bits, the shift in position will occur per bit basis.

4.4 OPTICAL TRANSMISSION LINK LIMITS

In overall, taking into account different impairments, the transmission system link length can be either power or bandwidth limited. Power limit means that different signal impairments (and dispersion among them) will be taken into account and will determine the total length. On the other side, in bandwidth limited case, the total capacity is determined by frequency bandwidth of the key components that are used (such as light source, photodetector, and optical fiber).
4.4.1.1 Power-Budget Limited Point-to-Point Lightwave Systems

There are two cases when considering power budget point-to-point lightwave systems: (i) link without optical amplifiers, (ii) link with a chain of the employed optical amplifiers. In case without optical amplifiers, the power budget is expressed as

$$P_{out} - \alpha L - \alpha_c - \Delta P_M \geq P_R(Q, \Delta f) + \Delta P_{imp}$$

(4.171)

where $P_{out}$ is output optical power from the light source pigtail, $\alpha$ is attenuation coefficient of the optical fiber, $\alpha_c$ is the signal loss related to optical splices and connectors, while $P_R$ is the receiver sensitivity related to specified BER. Parameter $\Delta P_M$ is the system margin that is needed to account for different operational effects such as aging, and temperature change, while $\Delta P_{imp}$ is power margin to account for the impact of impairments that are relevant for that specific case. Please notice that $P_R$ is expressed as a function of $Q$-factor and the receiver bandwidth (or the signal bit rate). All parameters in Equation (4.171) are expressed in decibels, with except to attenuation coefficient that is expressed in dB/km.

We can differentiate several cases here, based on what parameters are specified in advance. The common case occurs if both the total transmission distances $L$ and bit rate $B$ are specified, in which situation Equation (4.171) should help to select cost effective components that satisfy system requirements. This case is common for lower bit rates. The selection includes sources, operating wavelength, and the optical receiver. Generally speaking, all components are cheaper if they operate at shorter wavelengths. The lowest price is for components operating around 850 nm, and increases when shifting to wavelengths around 1310 nm and 1550 nm. As for light sources, LED-s are much cheaper than the laser photodiodes, while Fabry Perot lasers are cheaper than single mode DFB lasers. On receiver side, the APD-based receivers offer higher sensitivity than PIN-based receivers, however they need a high voltage supply that should be carefully controlled to avoid avalanche breakdown. In addition, an APD is more expensive than PIN, and that applies even if APD is compared with an integrated combination of PIN and FET based front-end amplifiers. Therefore, the selection process should go from LED to lasers, and from PIN photodiodes to APD, while checking if the selected component can satisfy system requirements. If there is a choice to select the optical fibers, the process should go from multimode to single-mode optical ones.

Equation (4.171) is mostly applicable bit rates up to 1 Gb/s and distances measured by tens of kilometers. As an example, let us consider case when transmission of the signal with bit rate of 400 Mb/s should be done over the distance that is not shorter than 15 km (Please refer to Table 1.1 to recognize that this bit rate corresponds to Fiber Channel-II.) We can assume that system is required to satisfy that $BER < 10^{-12}$. Let us start with LED as a source candidate.
and with both PIN and APD as photodetector candidates. We will assume that $P_{out} = -12 \text{ dBm}$, and that receiver sensitivities for PIN photodiodes and APD are $-30 \text{ dBm}$ and $-38 \text{ dBm}$, respectively. In addition, we can start with a multimode optical fiber and assume that the optical fiber bandwidth of a 1 kilometer optical fiber is $1.8 \text{ GHz} \cdot \text{km}$ (i.e. $B_{fib,L} = 1.8 \text{ GHz} \cdot \text{km}$)—please see Equation (4.95). Let us first consider the cost effective option (wavelength 850 nm), and the second choice (wavelength 1300 nm.)

We will assume that system margin of 5 dB can cover all operational requirements and signal impairments. It is a common approach to allocate the system margin in the range 4-6 dB [25] to cover for components aging and temperature effects. Finally, it is common to assume that splicing losses are included in fiber attenuation coefficient ($\alpha = 3.0 \text{ dB/km}$ at 850 nm, and $\alpha = 0.5 \text{ dB/km}$ for 1300 nm), while total connector losses are 2 dB. This situation is illustrated in Table 4.6.

We can clearly see that transmission with neither combination operating at 850 nm is viable (it shown in italic in Table 4.6). The next available option is to use transmission at 1300 nm, where the LED/PIN combination is the cheapest one that still satisfies requirements. If there were request to transmit signal over 22 km, the combination LED/APD operating at 1300 nm could be deployed. Any requirement for transmission over 36 km can be satisfied only by using the laser diodes operating at 1300 nm.

### Table 4.6

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LED</th>
<th>Laser</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>850 nm</td>
<td>1300 nm</td>
</tr>
<tr>
<td>Receiver sensitivity - PIN</td>
<td>-30 dB$_m$</td>
<td>-31 dB$_m$</td>
</tr>
<tr>
<td>Receiver sensitivity - APD</td>
<td>-38 dB$_m$</td>
<td>-39 dB$_m$</td>
</tr>
<tr>
<td>Output power</td>
<td>-12 dB$_m$</td>
<td>-13 dB$_m$</td>
</tr>
<tr>
<td>Connector losses</td>
<td>2 dB</td>
<td>2 dB</td>
</tr>
<tr>
<td>System margin</td>
<td>5 dB</td>
<td>5 dB</td>
</tr>
<tr>
<td>Available loss for PIN</td>
<td>11 dB</td>
<td>11 dB</td>
</tr>
<tr>
<td>Available loss for APD</td>
<td>19 dB</td>
<td>19 dB</td>
</tr>
<tr>
<td>Fiber loss per km</td>
<td>3.5 dB/km</td>
<td>0.5 dB/km</td>
</tr>
<tr>
<td>Transmission length for PIN</td>
<td>3.2 km</td>
<td>22 km</td>
</tr>
<tr>
<td>Transmission length for APD</td>
<td>5.4 km</td>
<td>36 km</td>
</tr>
</tbody>
</table>

In addition to the system margin, a power margin to compensate for the impact of the modal noise and mode partition noise should also be allocated if transmission is performed over multimode optical fibers while using laser as a light source. The margin of 1-2 dB can serve for that purpose. In case presented in Table 4.6, we assumed that 4 dB was allocated as system margin, and the additional 1 dB was allocated to compensate for the impact of the modal noise and mode partition noise. Presented analysis could be applied for the LAN environment.
The selection of transmission wavelength in case presented above may be related to the wavelength availability, since it might happen that wavelength that was originally selected might be already occupied. If such scenario occurs, some other wavelengths should be considered. Optical transmission in the local area network (LAN) environment often includes signal broadcast by using optical power splitting. In such a case, each optical 1:2 coupler/splitter employed along the lightwave path should be accounted by allocating 3 dB power-splitting loss. As an example, if there are 5 optical couplers, additional 15 dB should be added to the connection losses in Table 4.8.

Another transmission type that might be the power-budget limited is related to higher bit rates and wavelengths around 1310 nm. For example, it applies to bit rates of 2.5/10/40 Gb/s if they need to be transmitted over distances ranging from couple hundred meters to several kilometers. This scenario includes a data center networking (which is discussed in Chapter 8). The analysis related to this case is similar to one presented in Table 4.8, but with different starting points since only lasers should be considered as light source. Both the Fabry-Perot (FP) lasers and VCSEL are good candidates for bit rates up to 2.5 Gb/s, while VCSEL and the DFB lasers should be considered for bit rates up to 10 Gb/s. As for 40 Gb/s bit rates, DFB lasers monolithically integrated with electroabsorption modulators should be considered as primary candidates. As for the power margin allocation with respect to this transmission scenario, the power penalties related to the intensity noise and the extinction ratio impact should be covered by some power margin that comes in addition to system power margin. The power margin of up to 1 dB for each of these impairments is needed. The situation that might apply to high bit rates is illustrated in Table 4.7. In most cases, however, that transmission distances is not power budget-limited, but rather bandwidth-limited, as we will see shortly, so the values presented in Table 4.7 may not be relevant.

### Table 4.7

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2.5 Gb/s</th>
<th>10 Gb/s</th>
<th>40 Gb/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength in nm</td>
<td>1310</td>
<td>1310</td>
<td>1310</td>
</tr>
<tr>
<td>Receiver sensitivity for PIN, in dBm</td>
<td>-27</td>
<td>-24</td>
<td>-21</td>
</tr>
<tr>
<td>Receiver sensitivity for APD, in dBm</td>
<td>-35</td>
<td>-32</td>
<td>-29</td>
</tr>
<tr>
<td>Output power in dBm</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Connector losses in dB</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>System margin in dB</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Additional margin in dB</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Available loss for PIN in dB</td>
<td>15</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>Available loss for APD in dB</td>
<td>23</td>
<td>20</td>
<td>14</td>
</tr>
<tr>
<td>Fiber loss in dB/km</td>
<td>0.5</td>
<td>0.22</td>
<td>0.5</td>
</tr>
<tr>
<td>Transmission length for PIN in km</td>
<td>30</td>
<td>24</td>
<td>12</td>
</tr>
<tr>
<td>Transmission length for APD in km</td>
<td>46</td>
<td>90</td>
<td>28</td>
</tr>
</tbody>
</table>
4.4.1.2 Bandwidth-Limited Point-to-Point Lightwave Systems

The performance of an optical transmission system can be limited due to available frequency bandwidth of some of the key components that are used (such as light source, photodetector, or optical fiber). The optical fiber is the most important from the system bandwidth perspective. Optical fiber bandwidth in multimode fibers is limited mainly due to modal dispersion effect since it is usually much larger than the chromatic dispersion, while the chromatic dispersion is the only factor that defines the bandwidth in single mode optical fibers.

The bandwidth of multimode optical fibers is characterized through the bandwidth $B_{fib}$ of the 1-km optical fiber length, which is somewhere around 150 MHz-km for step-index optical fibers, and around 2 GHz-km for well-designed graded-index optical fibers (otherwise it is below 1 GHz-km). As for single-mode optical fibers, the available bandwidth also depends on the spectral linewidth of the light source that is used. Generally, we can distinguish two separate cases with respect to the available bandwidth of single-mode fibers and light sources. We can use Equations (4.95), (3.143), and (3.145) to evaluate the available fiber bandwidth through the bandwidth-length product of any given length. Accordingly, we can distinguish the following three cases (one for multimode and two for single mode fibers), which is expressed as

$$BL^\mu \leq B_{fib} , \text{ for multimode optical fibers} \quad (4.172)$$

$$BL \leq (4D\sigma_\lambda)^{-1} , \text{ for single mode fibers and large source linewidth} \quad (4.173)$$

$$B^2L \leq (16B_2)^{-1} , \text{ for single mode fibers and narrow source linewidth} \quad (4.174)$$

where $B$ is signal bit rate, $L$ is the transmission length, $B_{fib}$ is the bandwidth of multimode optical fibers, $\mu$ is parameter that ranges from 0.5 to 1, $D$ is the chromatic dispersion coefficient, $\sigma_\lambda$ is the light source linewidth, and $B_2$ is group velocity dispersion coefficient for single mode optical fibers. Therefore, Equations (4.172-4.174) can be used in an initial assessment to see if the optical transmission system is power budget-limited or bandwidth-limited.

It is useful to plot the distance as function of the bit rate by using Equations (4.172-4.174). It can be done in parallel with the curves that are related to the optical power budget limitations. These curves can be plotted by using the following functional dependence derived from Equation (4.171)

$$L(\lambda, B) \leq \frac{P_{out}(\lambda, B) - \left[ P_R(\lambda, B) + \alpha_c + \Delta P_s + \Delta P_{mp} \right]}{\alpha(\lambda)} \quad (4.175)$$

where $P_{out}$ is output optical signal power, $P_R$ is receiver sensitivity, $\alpha_c$ is attenuation coefficient of the optical fiber, $\alpha$ is signal loss related to optical
splices and connectors, $\Delta P_M$ is the system margin. For this purpose we can assume that there is no system margin, and that connector losses can be neglected.

Functional curves expressed by Equations (4.172-4.175) are shown in Figure 4.23. The transmission distance is shown as a function of the bit rate, while optical wavelength serves as a parameter. Figure 4.23 can be used to identify the reference points, and to recognize what is a critical limitation factor. It was assumed that the GVD coefficient $\beta_2$ in single-mode fibers (SMF) is $\beta_2 = -20$ ps$^2$/km, while it is $\beta_2 = -4$ ps$^2$/km for nonzero-dispersion shifted fibers (NZDSF).

In general, there are several conclusions with respect to system limitations when using different fiber types. First, the systems with step-index multimode optical fibers are bandwidth-limited for all bit rates of practical interest (ranging from 1 Mb/s to several megabits per second), while the achievable distance is up to several kilometers. Secondly, the systems with graded-index multimode optical fibers are generally power budget-limited if the bit rate is up to about 100 Mb/s, while they become dispersion-limited for higher bit rates. It is possible to transfer 1Gb/s bit rate up to 1.5-2 km, or 10 Gb/s bit rate up to ~300 m over graded-index multimode optical fibers if intensity modulation and direct detection (IM/DD) scheme is applied. Next, single-mode optical fibers are also power budget-limited for bit rates up to several hundred Mb/s if optical source has a large linewidth. The power budget-limit can be extended to bit rates of 2.5 Gb/s and over if optical sources with a narrow linewidth are used. In addition, different types of single-mode optical fibers impose different kind of limits with respect to a specific bit rate. As an example, standard single mode optical fibers (SMF) impose the power-budget limit to signals with bit rates of 1 Gb/s if operation is done at wavelengths around 1300 nm, while non-zero dispersion shifted fibers impose the bandwidth limit for the this bit rate and the operational wavelength.

---

**Figure 4.23** System length limitations due to signal loss and fiber bandwidth
The operation over single mode fibers at wavelengths around 1,300 nm is the most advantageous for signals with bit rates up to 600 Mb/s. It is important to mention that it is also possible to transmit signal with bit rate of 2.5 Gb/s up to 50-60 km over SMF by applying direct modulation scheme at the wavelength of 1,300 nm. The same arrangement can be used to transmit signals with 10 Gb/s bit rate up to 10-15 km. Single mode optical fibers provide a dispersion-limited transmission at the wavelengths that belong to 1,550 nm wavelength region. With this arrangement, it is possible to transmit signals with 10 Gb/s bit rate up to 40-50 km, or signals with bit rate of 40 Gb/s up to 2-3 km. Please also see data from Table 4.3 about transmission lengths that can be achieved for specified bit rate without using any dispersion compensation scheme. (It can be also used to make an assessment of the amount of residual chromatic dispersion that can be tolerated in bandwidth-limited systems.)

The transmission length of the bandwidth-limited systems is not determined just by the optical fiber bandwidth expressed by Equations (4.172-4.174), but also by the frequency bandwidth (3-dB bandwidth) of the optical transmitter and optical receiver. The bandwidth limitation is related to the pulse rise times that occur in individual modules in the system, and the following relation can be established

\[ T_r^2 \geq T_w^2 + T_{fib}^2 + T_{rec}^2 \]  \hspace{1cm} (4.176)

where \( T_r \) is the overall response time of the system, \( T_w \) is the response time (i.e. the rise time) of the optical transmitter, while \( T_{rec} \) is the response time of the optical receiver. There is a following general relationship between the response time and the 3-dB system bandwidth \( \Delta f \) [1], [25]

\[ T_r = \frac{0.35}{\Delta f} \]  \hspace{1cm} (4.177)

Above equation can be converted to a general form that connects the response time with the signal bit rate. The relation between system bandwidth \( \Delta f \) and bit rate \( B \) is dependent on the digital format that is used. It is \( \Delta f = B \) if return-to-zero (RZ) binary modulation format is used, while it is \( \Delta f = 0.5B \) if the signal has non-return-to-zero (NRZ) binary format. Connection between the bandwidth and the rise time expressed by Equation (4.177) can be applied to all key elements in the optical system (i.e. to optical transmitter, optical fiber, and optical receiver). With this, Equation (4.177) becomes

\[ \frac{1}{B^2} \geq \frac{1}{\Delta f^2_w} + \frac{1}{B_{fib,L}^2} + \frac{1}{\Delta f_{rec}^2}, \quad \text{for RZ format} \]  \hspace{1cm} (4.178)
where \( B_{fib,L} \) is the optical fiber bandwidth related to specified length \( L \), which can be calculated from Equations (4.172-4.174). Please notice that Equations (4.172-4.174) should be solved first in order to obtain the value of parameter \( B \) that is related to specified transmission length \( L \). The obtained value is than assigned to the fiber bandwidth \( B_{fib,L} \).

Equations (4.178-4.179) can be used to verify whether the system is bandwidth-limited or not. They should be solved to find the transmission length \( L \), while the obtained result should be compared with the value obtained from Equation (4.171), which is related to the power budget-limit scenario. Assuming that the transmitter and receiver satisfy bandwidth requirement, the result of comparison will be determined by the optical fiber type and by operational wavelength. However, if it happens that transmitter and receiver do not satisfy requirements given by Equations (4.178-4.179), faster alternatives should be considered.

It is important to mention that all considerations, presented in this subsection with respect to binary IM/DD scheme, can be applied to more multilevel modulation schemes, where symbol rate would be used instead of bit rate with an adjustment of bandwidth \( \Delta f \) to detection scheme that is applied.

4.4.1.3 OSNR Evaluation in High-Speed Optical Transmission Systems

Results related to point-to-point transmission can be generalized for optically amplified system that employs some number of optical amplifiers. The Equation (4.171) can be now replaced by Equation (4.167), which can be rewritten as

\[
\frac{4}{B^2} \geq \frac{1}{\Delta f_{s}^2} + \frac{1}{B_{ch}^2} + \frac{1}{\Delta f_{rec}^2}, \quad \text{for NRZ format} \quad (4.179)
\]

Please notice that a power margin \( \Delta P \) has been introduced to account for the impact of various impairments. The starting point when applying Equation (4.180) will be again a specified BER, which is related to the \( Q \)-factor. The entrance parameters are the signal/symbol bit rate and the number of optical channels. Equation (4.180) can also be written as

\[
\text{OSNR} \approx \frac{2Q^2\Delta f}{B_{op}} \geq P_{ch} - NF_{no} - \alpha L_{span} - 10\log(N) - 10\log(h\nu B_{op}) - \Delta P \quad (4.180)
\]

where \( OSNR_{req} \) is a new optical signal-to-noise-ratio that should be achieved in order to accommodate power penalties imposed by various impairments. It is clear that this value should be higher than the original one by amount equal to power penalties, which are expected to occur. Equation (4.181) is the basic one that can
be used in design process with respect to high-speed long-haul transmission systems. The parameters that are included into considerations are the launched output power per channel, span length, number of channels, channel spacing, and number of spans. The launched output power is a parameter that should be optimized by evaluating merits and demerits of its increase, which means that the power level can be increased only to the point where increase in OSNR is higher than the power penalty imposed by nonlinear effects. The number of optical channels will have an impact to nonlinear effects and crosstalk noise. As for the fiber span length, it is commonly predetermined, and the system considerations are than based on evaluation of the maximum number of spans that can be accommodated under specified conditions. The $\text{OSNR}_{\text{req}}$ has been evaluated in [33] and the values of 15.02 dB, 20.12 dB, and 22.70 dB have been obtained for bit rates of 2.5 Gb/s, 10 Gb/s and 40 Gb/s, respectively, in case when IM/DD scheme is applied and with $Q$-factor equal 7.

Margin assignment plays very important role in the system design process. The assignment may be based on a conservative approach, in which case the total margin is a sum of individual contributions. There are several impairments, such as chromatic dispersion and the extinction ratio that do not have a stochastic character and require allocation of a quite specific margin. On the other side, there are some impairments, such as PMD and intensity noise, that are stochastic in nature and can combine differently with each other at any given moment. These impairments can be covered by a joint power margin, which is allocated for all of them. The value of the margin can be estimated by applying a statistical approach that evaluates a joint impact of several parameters. The simplest option is to consider these parameters to be random Gaussian variables, and to assign the margin that is proportional to the total standard deviation of the summary stochastic process. The alternative approach is to use statistical modeling, such as one that employs the Monte-Carlo method, in order to get the most realistic outcome. In such a case, the margin allocation will be based on the results of the most realistic outcome. The main benefit from statistical approach is that the allocated margin is not overvalued, as it might be if applying a conservative scenario. Finally, the margin allocation can be based on computer-aided engineering and use of specialized simulation software. This will lead to more complex calculations, and will provide a better estimate of some important effects that are otherwise rather difficult for evaluation (such as cross phase modulation).

4.5 SUMMARY

In this chapter we explained the origin of the noise components in an optical channel and evaluated their relevance. We also evaluated the impact of the various linear and nonlinear effects that contribute to the signal distortion and receiver sensitivity degradation. The special attention has been be paid to evaluation of the quantum shot noise, amplified spontaneous emission impact, and laser intensity
and phase noise components. Also, a detailed assessment of the impact of the pulse broadening due to chromatic dispersion, polarization mode dispersion, and selfphase modulation has been performed. The impairment analysis in multichannel transmission included linear crosstalk, crossphase modulation, four-wave mixing, and stimulated Raman scattering. Finally, transmission system performance has been evaluated under two scenarios determined by the system power budget and by the total pulse broadening.

Chapter 4: Problems

4.1 Draw a block diagram of a noise created in optical communication channel and in digital optical receiver, showing its various components. Explain the origin of each component.

4.2 What is difference between additive and multiplicative noise components? Name the most important additive and multiplicative components.

4.3 What is probability that one electron-hole pair will be created within some time interval if the average number of the electron-hole pairs in such interval is also one. What would be probability to have 10 pairs of electron holes within the same time period?

4.4 Optical receiver has InGaAsP PIN photodiode operating at 1300 nm. It has 50 MHz bandwidth, 70% quantum efficiency, 2 nA dark current, 10 pF junction capacitance, and 3 dB amplifier noise figure. There is optical power of $10^{-6}$ W coming to PIN. Find the spectral densities, as well as the variances of the noise currents due to shot noise, dark current, thermal noise, and amplifier noise.

4.5 In problem 4.2 calculate SNR and receiver sensitivity.

4.6 What is the minimum received power of the receiver from Problem 4.4 when the detection is limited by (a) shot noise and (b) thermal noise if required Q factor is 25 dB.

4.7 Calculate maximum Q-factor if APD is used, and find the optimum value M of the APD gain. Use expression $F(M) = M^x (x=0.7)$ for APD noise factor. Assume that receiver sensitivity is -30 dBm, receiver bandwidth is 100 MHz, and photodiode responsivity is 0.8 A/W. Load resistance has value of 100 Ω, while noise figure of frontend amplifier is 3.

4.8 Find receiver sensitivity at BER= $10^{-9}$ for a PIN based digital receiver operating 1.3 μm at bit rate of 200 Mb/s, if receiver bandwidth is 100 MHz. PIN has 80% quantum efficiency, the load resistance is 100 Ω and the frontend amplifier noise figure is 3 dB. How much additional power is needed to satisfy requirement of BER= $10^{-15}$? How many photons are needed in both cases? Is system thermal noise or shot noise limited?

4.9 Calculate sensitivity referred to BER= $10^{-9}$ of an APD receiver has been degraded by 2 dB due to finite extinction ratio, in the case where both avalanche shot noise and thermal noise are present. The APD has the following parameters: M=10, x=0.7, and quantum efficiency 70%. Assume
that receiver bandwidth is 300 MHz, and that noise figure of the front-end amplifier is 3. What is extinction ratio that would guarantee that Q-factor degradation is less than 30%.

4.10 The impact of nonideal extinction ratio to receiver sensitivity of APD based optical receivers can be evaluated if we assume that the quantum shot noise is dominant noise factor. Compare degradations in receiver sensitivity versus degradations in Q-factor (use decibel units). Use APD parameters from Problem 4.9 if necessary. Extinction ratio can change from 5 to 50. Plot functional curves reflecting dependence on the nonideal extinction ratio.

4.11 Optical receiver has InGaAsP alloy PIN photodiode operating at 1,550 nm. The receiver has 5 GHz bandwidth, 70% quantum efficiency, 0.5 pF junction capacitance, and 3 dB amplifier noise figure. There is optical power of 1 \( \mu \)W at 10 Gb/s coming to PIN. There is also local optical oscillator (LO) deployed in heterodyne detection scheme with power of 10 dBm, and RIN (0) = -160 dB. Find the standard deviations of the noise currents due to shot noise, thermal noise, and laser intensity noise. What power of LO would equalize contributions from shot noise on one side and intensity noise plus thermal noise on the other side?

4.12 Provide the power budget and estimate the link length of 1310 nm long-haul lightwave system that operates at 2.5 Gb/s. The average power of 0.5 mW is coupled into the fiber. The 0.45-dB/km fiber-cable loss includes splice losses. The connectors at each end have 1-dB losses. The InGaAs PIN receiver has a sensitivity of -36 dBm.

4.13 With respect to Equation (4.177), confirm that factor 0.35 has the origin in 3-dB bandwidth of RC circuits.

4.14 What would be the factor used in Equation (4.177) if the Gaussian optical pulse with the rise time (with respect to the 3-dB optical bandwidth) is used as a reference?

4.15 Optical transmission system operates at bit rate of 100 Mb/s. Make the rise-time budget is transmission is done at wavelength 1.31 \( \mu \)m over 15 km. The LED that has been used in transmitter, and the InGaAs PIN used in receiver have rise times of 3 and 1.7 ns, respectively. Transmitted power is equal to -13 dBm, while receiver sensitivity is equal -34 dBm. The graded-index multimode fiber has a core index of \( n_c = 1.45 \), relative core-cladding index difference \( \Delta = 0.01 \), and chromatic dispersion parameter \( D = 0.5 \text{ ps/(km-nm)} \). The LED spectral width is 30 nm. Is system power budget limited? Can the system be designed to operate with both NRZ and RZ formats? Plot the system length limitations with respect to both approaches.

4.16 Optical transmission system operates at 1,530 nm with bit rate of 2.5 Gb/s over the fiber line with OEO repeater spacing of 40 km. The fiber is DSF with a chromatic dispersion of 1.3 ps/(km-nm) in the vicinity of the operating wavelength. Calculate the wavelength linewidth of multimode semiconductor lasers for which the impact of mode-partition-noise can be almost entirely suppressed. Assume that mode-partition coefficient \( k = 0.7 \).
4.17 Find the maximum transmission distance for digital transmission systems operating at 8 Gb/s to keep the chirp-induced power penalty below 1 dB. Single mode laser with chirp \(C_0 = -4\) is used over single-mode fiber with \(\beta_2 = -20\) ps\(^2\)/km.

4.18 What is noise figure of an optical amplifier? Explain it for common cases (EDFA and Raman optical amplifiers). Why noise figure can not be lower than 3 dB.

4.19 Optical transmission system use EDFA as a preamplifier. Calculate the receiver sensitivity at BER\(= 10^{-12}\) and BER\(=10^{-15}\). Assume that carrier wavelength is 1,610 nm and that receiver bandwidth is 7 GHz. The noise figure of the preamplifier is 5 dB, and a 1-nm optical filter is installed between the preamplifier and the detector. What is receiver sensitivity if receiver bandwidth is equal to the filter bandwidth?

4.20 Submarine optical system 10000 km long is designed to operate with bit rate of 10 Gb/s at carrier wavelength of 1550 nm. What is \(Q\) factor if EDFA are places every 50 km? Assume a fiber-cable loss o is 0.22 dB/km, launched power is 0 dBm, NF of EDFA is 4 dB and a 2-nm-bandwidth optical filter is inserted after every amplifier to reduce ASE noise. Is this good quality transmission assuming that FEC will be applied that needs \(Q\) to be more than \(10^{-3}\)?

4.21 There are 5 EDFA on the line with the following: G= 23, NF= 7; G= 20, NF=6; G=19, NF=6; G =25, NF=4.6. Provide the best alignment to have maximum SNR if input signal is 1 microwatt.

4.22 Calculate OSNR for Problem 4.20 if optical filter with a bandwidth of 100 GHz is applied.

4.23 What is required OSNR to achieve BER\(<10^{-15}\) in transmission of 40 Gb/s by using RZ modulation. Assume that you can select the optical filter characteristics.

4.24 What is the dispersion-limited transmission distance for 10 Gb/s RZ directly modulated lightwave system operating at 1.55 \(\mu\)m over standard SMF? Assume that frequency chirping broadens the Gaussian shape pulse spectrum by factor 4 from its original form. What would be improvement if external modulator with chirp factor equal -1 is used instead of direct modulation?

4.25 The external modulation with adjustable chirp parameter is used in 10-Gb/s lightwave system operating at 1,550 nm over standard SMF, while transmitting RZ bits as chirped Gaussian pulses of 40 ps width (FWHM). Dispersion penalty up to 1 dB can be tolerated. What is the optimum value of the chirp parameter \(C_0\), and how far can the signal be transmitted for this optimum value? What would be distance if 2 dB penalty is tolerated?

4.26 What would be the impact to solution of Problem 4.17 if optical filter with the transfer function given by \(H(\omega) = exp\{-1 + j\omega\}\) is used? Hint: find the impulse response of this filter. Use Equation (3.186) to find the pulse shape at the filter output. How would you optimize the filter to minimize the effect of fiber dispersion?
4.27 What is difference in achievable length to keep dispersion penalty below 1 dB if the source with large spectral linewidth is used, and if an external modulation is applied to CW laser output. If PMD DGD is 0.07 ps/(km)^{1/2}, find the distance and compare it with the chromatic dispersion limit. Make conclusions.

4.28 Find maximum DGD and the maximum transmission distance to keep receiver sensitivity penalties within 1 dB limit for 40 Gb/s binary IM/DD system. Assume that there is also second order PMD and that impact of all elements is equal to the impact of the second order. What is difference in case when QPSK modulation format is applied?

4.29 Derive an expression for the power penalty induced by crosstalk in a waveguide-grating router. Calculate the power penalty if leaking is 0.1 %.

4.30 Calculate the local oscillator power employed in heterodyne receiver to suppress the impact of thermal noise (thermal noise to be less than 1% of the total noise). Assume operation at room temperature. Receiver employs PIN photodiode of 90% quantum efficiency, which is connected to a 50 Ω load resistance.

4.31 There is chain of optical amplifiers spaced 80 km apart in systems operating at 25 Gb/s at 1,550 nm over SMF line. Each amplifier has NF=5 dB, and gain 20 dB. Find total length of the line to have BER= 10^{-12} is RZ coding is employed. Assume that optical bandwidth can be optimized, and system margin is 5 dB. There is 80 channels and XPM, as the most critical impairment, should be kept under control.

4.32 We would like to design an optical 160 Gb/s WDM system with M=4 channels operating at 40 Gb/s employing the hybrid Raman-EDFA in-line optical amplifiers, as shown in figure below. The total launched power is restricted by available laser sources, and cannot be larger than 9 dBm, and the SMF fiber loss is α=0.2 dB/km. SMF dispersion coefficient is 17 ps/nm-km. Dispersion compensating module (DCM) is composed of DCF with dispersion coefficient of -95 ps/nm-km, while DCF attenuation coefficient is 0.5 dB/km. Notice that DCF length does not contribute to the total transmission length. Distributed Raman amplifier (DRA) is used to compensate for SMF fiber loss, while EDFA is used to compensate for DCM insertion loss. Assuming that double Rayleigh backscattering can be neglected the DRA noise figure can be calculated by $NF_{DRA}=(P_{ASE}/h\nu B_{op}+1)/(G_{DRA}e^{-\alpha l})$, where $G_{DRA}$ is DRA gain ($B_{op}$ is the optical filter bandwidth). The ASE noise power of DRA can be calculated by $P_{ASE}=2h\nu \Delta \nu r/(1-exp(-h\nu/kB\theta))$, $kB$ is the Boltzmann constant (1.381x10^{-23} J/K), $\nu s$ is the signal frequency, $\Delta \nu r$ is the separation between signal and pump frequencies. The pump wavelength is 1450 nm. $\theta$ is the absolute temperature. Assume that $G_{DRA}$ is just enough to compensate for SMF fiber loss. The EDFA noise figure ($NF_{EDFA}$) is 6 dB. The equivalent noise figure of both DRA and EDFA can be calculated by $NF=NF_{DRA}+(NF_{EDFA}+1)/(G_{DRA}e^{-\alpha l})+1-\Gamma$, where $\Gamma$ is loss caused by DCM. The predicted
margins to achieve $BER=10^{-12}$ allocated for chromatic dispersion, extinction ratio, polarization effects, fiber nonlinearities, component aging, and system margin are 0.5, 0.5, 1.5, 1.5, 2, and 2 dB respectively. The operating temperature is 25°C. WDM DEMUX can be modeled as a bank of optical filters, each of them with a bandwidth $B_o$ equal to $B$ ($B$-the bit rate per channel). Assuming NRZ transmission, the electrical filter bandwidth $B_e=0.7B$ and amplifier spacing $l=100km$ determine the required optical signal-to-noise ratio (OSNR) to achieve $BER$ of $10^{-12}$ for the central channel located at 1552.524nm. What is the maximum possible transmission distance? The saturation of EDFAs due to ASE noise accumulation is negligible.

![Diagram](image)

References


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